

Pre-Calculus

Techniques for Finding Limits – Day 1

Name:

Date:

Period:

Find the limit if it exists. Try these techniques in this order:

- ☉ Direct Substitution
- ☉ Algebra (factoring/simplifying or rationalizing the numerator or adding the fractions to simplify)
- ☉ Find it graphically by looking at the graph.
- ☉ Find it numerically by looking at a table.

$$1) \lim_{x \rightarrow 0} \frac{x^3 - x}{x - 1} = \frac{0^3 - 0}{0 - 1} = \frac{0}{-1} = 0$$

$$2) \lim_{x \rightarrow -1} \frac{x^3 - x}{x - 1} = \frac{(-1)^3 - (-1)}{-1 - 1} = \frac{-1 + 1}{-2} = \frac{0}{-2} = 0$$

$$3) \lim_{x \rightarrow 1} \frac{x^3 - x}{x - 1} = \frac{1^3 - 1}{1 - 1} = \frac{0}{0}$$

$$\lim_{x \rightarrow 1} \frac{x(x^2 - 1)}{x - 1} = \frac{1(1+1)}{1-1} = \frac{2}{0} = \text{DNE}$$

$$4) \lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x - 1} = \frac{1^2 + 2 - 3}{1 - 1} = \frac{0}{0}$$

$$5) \lim_{x \rightarrow 6} \frac{x - 6}{x^2 - 36} = \frac{6 - 6}{6^2 - 36} = \frac{0}{0}$$

$$6) \lim_{x \rightarrow 6} \frac{x + 6}{x^2 - 36} = \frac{12}{6^2 - 36} = \frac{12}{0} = \text{DNE}$$

$$\lim_{x \rightarrow 0} \frac{(x+3)(x-1)}{x-1} = 0 + 3 = 3$$

$$\lim_{x \rightarrow 6} \frac{(x-6)}{(x+6)(x-6)} = \lim_{x \rightarrow 6} \frac{1}{x+6} = \frac{1}{12}$$

$$7) \lim_{x \rightarrow 0} \frac{\sqrt{x+3} - \sqrt{3}}{x} = \frac{\sqrt{3} - \sqrt{3}}{0} = \frac{0}{0}$$

$$8) \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$$

$$9) \lim_{x \rightarrow 0} \frac{\frac{1}{x-8} + \frac{1}{8}}{x} = \frac{-\frac{1}{8} + \frac{1}{8}}{0} = \frac{0}{0} = \text{DNE}$$

$$\lim_{x \rightarrow 0} \frac{(\sqrt{x+3} - \sqrt{3})(\sqrt{x+3} + \sqrt{3})}{x(\sqrt{x+3} + \sqrt{3})} = \lim_{x \rightarrow 0} \frac{x+3-3}{x(\sqrt{x+3} + \sqrt{3})} = \frac{7}{8} \text{ on paper}$$

$$10) \lim_{x \rightarrow 0} \frac{\sec x}{\tan x}$$

$$11) \lim_{x \rightarrow 0} \frac{\frac{1}{x+1} - 1}{x}$$

$$12) \lim_{x \rightarrow 2} \frac{4 - \sqrt{18-x}}{x-2}$$

$$13) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x - 1}{\sin x}$$

$$14) \lim_{x \rightarrow \pi} \frac{\csc x}{\cot x}$$

$$15) \lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x}$$

$$16) \lim_{x \rightarrow 0} \frac{1 - e^{-x}}{x}$$

$$17) \lim_{x \rightarrow 0} (1-x)^{\frac{2}{x}}$$

$$18) \lim_{x \rightarrow 1} \frac{1 - \sqrt[3]{x}}{x-1}$$

$$19) \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x}$$

$$20) \lim_{x \rightarrow 2} \frac{x^4 - 1}{x^4 - 3x^2 - 4}$$

$$21) \lim_{x \rightarrow 0} x^2 \sin x^2$$

$$22) \lim_{x \rightarrow 0} \frac{x}{\cos x}$$

$$23) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

$$24) \lim_{x \rightarrow 0} \frac{\sin x^2}{x^2}$$

$$7. \lim_{x \rightarrow 0} \frac{1}{x(\sqrt{x+3} + \sqrt{3})} = \frac{1}{\sqrt{3} + \sqrt{3}} = \boxed{\frac{1}{2\sqrt{3}}}$$

$$8. \lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} = \frac{\sqrt{4} - 2}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{(\sqrt{x+4} - 2)(\sqrt{x+4} + 2)}{x(\sqrt{x+4} + 2)} = \lim_{x \rightarrow 0} \frac{x+4-4}{x(\sqrt{x+4} + 2)} = \lim_{x \rightarrow 0} \frac{1}{x(\sqrt{x+4} + 2)}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+4} + 2} = \frac{1}{\sqrt{4} + 2} = \boxed{\frac{1}{4}}$$

$$9. \lim_{x \rightarrow 0} \frac{1 \cdot 8}{(x-8)^8} \cdot \frac{1}{8(x-8)} \lim_{x \rightarrow 0} \frac{8+x-8}{8(x-8)} = \lim_{x \rightarrow 0} \frac{x}{8(x-8)}$$

$$= \lim_{x \rightarrow 0} \frac{x}{8(x-8)} \cdot \frac{1}{x} = \lim_{x \rightarrow 0} \frac{1}{8(x-8)} = \frac{1}{8(-8)} = \boxed{-\frac{1}{64}}$$

$$10. \lim_{x \rightarrow 0} \frac{\sec x}{\tan x} = \lim_{x \rightarrow 0} \frac{1}{\frac{\sin x}{\cos x}} = \lim_{x \rightarrow 0} \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sin x} = \frac{1}{\sin 0} = \frac{1}{0} \quad \boxed{\text{DNE}}$$

$$11. \lim_{x \rightarrow 0} \frac{\frac{1}{x+1} - \frac{x+1}{x+1}}{x} = \lim_{x \rightarrow 0} \frac{1 - (x+1)}{\frac{x+1}{x}} = \lim_{x \rightarrow 0} \frac{-x}{x+1} \cdot \frac{1}{x}$$

$$= \lim_{x \rightarrow 0} \frac{1}{x+1} = \frac{1}{1} = \boxed{1}$$

$$12. \lim_{x \rightarrow 2} \frac{4 - \sqrt{18-x}}{x-2} = \frac{4 - \sqrt{18-2}}{2-2} = \frac{4 - \sqrt{16}}{0} = \frac{4-4}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 2} \frac{(4 - \sqrt{18-x})(4 + \sqrt{18-x})}{(x-2)(4 + \sqrt{18-x})} = \lim_{x \rightarrow 2} \frac{16 - (18-x)}{(x-2)(4 + \sqrt{18-x})}$$

$$= \lim_{x \rightarrow 2} \frac{-2+x}{(x-2)(4 + \sqrt{18-x})} = \lim_{x \rightarrow 2} \frac{-(x-2)}{(x-2)(4 + \sqrt{18-x})} = \frac{-1}{4 - \sqrt{16}} = \frac{-1}{0}$$

DNE

$$13. \lim_{x \rightarrow \pi/2} \frac{\cos x - 1}{\sin x} = \frac{\cos \pi/2 - 1}{\sin \pi/2} = \frac{0 - 1}{1} = \boxed{-1}$$

$$14. \lim_{x \rightarrow \pi} \frac{\csc x}{\cot x} = \lim_{x \rightarrow \pi} \frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}} = \lim_{x \rightarrow \pi} \frac{1}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{\cos x}$$

$$= \lim_{x \rightarrow \pi} \frac{1}{\cos x} = \frac{1}{\cos \pi} = \frac{1}{-1} = \boxed{-1}$$

$$15. \lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x} = \frac{e^0 - 1}{0} = \frac{0}{0} \quad \text{use calculator}$$

$$\lim_{x \rightarrow 0} \frac{e^{3x} - 1}{x} = \boxed{3}$$

$$16. \lim_{x \rightarrow 0} \frac{1 - e^{-x}}{x} = \frac{1 - e^{-0}}{0} = \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{1 - \frac{1}{e^x}}{x} = \lim_{x \rightarrow 0} \frac{\frac{e^x - 1}{e^x}}{x} = \lim_{x \rightarrow 0} \frac{e^x - 1}{e^x \cdot x} = \frac{0}{0} \quad \text{Use the calc.}$$

$$\lim_{x \rightarrow 0} \frac{1 - e^{-x}}{x} = \boxed{1}$$

$$17. \lim_{x \rightarrow 0} (1-x)^{2/x} \quad (1-0)^{2/0} \quad \text{use calc}$$

$$\lim_{x \rightarrow 0} (1-x)^{2/x} = \boxed{\approx 0.13534}$$

$$18. \lim_{x \rightarrow 1} \frac{1 - \sqrt[3]{x}}{x-1} = \frac{1-1}{1-1} = \frac{0}{0} \quad \text{use calc}$$

$$\lim_{x \rightarrow 1} \frac{1 - \sqrt[3]{x}}{x-1} = \boxed{-\frac{1}{3}}$$

$$19. \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x} = \frac{1-1}{0} = \frac{0}{0} \quad \text{use calculator}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x} = \boxed{0}$$

$$20. \lim_{x \rightarrow 2} \frac{x^4 - 1}{x^4 - 3x^2 - 4} = \frac{2^4 - 1}{2^4 - 3 \cdot 2^2 - 4} = \frac{16 - 1}{16 - 12 - 4} = \frac{15}{0} \quad \boxed{\text{DNE}}$$

$$21. \lim_{x \rightarrow 0} x^2 \sin x^2 = 0^2 \cdot \sin 0^2 = \boxed{0}$$

$$22. \lim_{x \rightarrow 0} \frac{x}{\cos x} = \frac{0}{\cos 0} = \frac{0}{1} = \boxed{0}$$

$$23. \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1 - \cos 0}{0^2} = \frac{1-1}{0} = \frac{0}{0} \quad \text{use calculator}$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \boxed{\frac{1}{2}}$$

$$24. \lim_{x \rightarrow 0} \frac{\sin x^2}{x^2} = \frac{\sin 0^2}{0^2} = \frac{0}{0} \quad \text{use calculator}$$

$$\lim_{x \rightarrow 0} \frac{\sin x^2}{x^2} = \boxed{1}$$

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