(b) Similarly

$$x_1 \cdot x_2 = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$

$$= \frac{b^2 - (b^2 - 4ac)}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$

81. $f(x) = (x - a)(x - b) = x^2 - bx - ax + ab$ $= x^2 + (-a - b)x + ab$. If we use the vertex form of a quadratic function, we have $h = -\left(\frac{-a-b}{2}\right)$ $= \frac{a+b}{2}$. The axis is $x = h = \frac{a+b}{2}$.

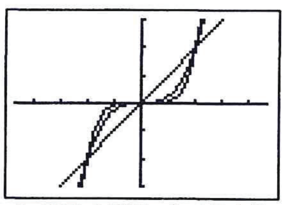
82. Multiply out $f(x)$ to get $x^2 - (a + b)x + ab$. Complete the square to get $\left(x - \frac{a+b}{2}\right)^2 + ab - \frac{(a+b)^2}{4}$. The vertex is then (h, k) where $h = \frac{a+b}{2}$ and $k = ab - \frac{(a+b)^2}{4} = -\frac{(a-b)^2}{4}$.

83. x_1 and x_2 are given by the quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; then $x_1 + x_2 = -\frac{b}{a}$, and the line of symmetry is $x = -\frac{b}{2a}$, which is exactly equal to $\frac{x_1 + x_2}{2}$.

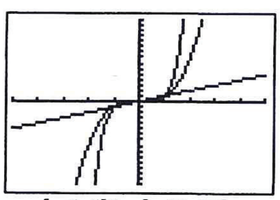
Section 2.2 Power Functions with Modeling

Exploration 1

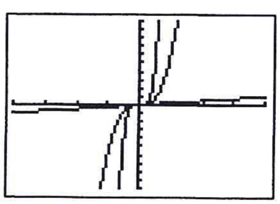
1.



[−2.35, 2.35] by [−1.5, 1.5]



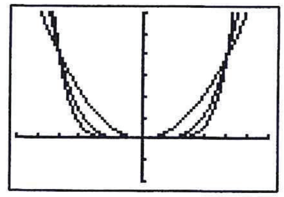
[−5, 5] by [−15, 15]



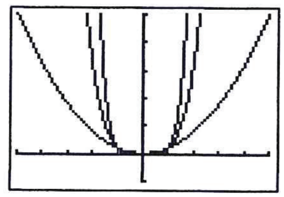
[−20, 20] by [−200, 200]

The pairs $(0, 0)$, $(1, 1)$, and $(-1, -1)$ are common to all three graphs. The graphs are similar in that if $x < 0, f(x), g(x)$, and $h(x) < 0$ and if $x > 0, f(x), g(x)$, and $h(x) > 0$. They are different in that if $|x| < 1$, $f(x), g(x)$, and $h(x) \to 0$ at dramatically different rates, and if $|x| > 1, f(x), g(x)$, and $h(x) \to \infty$ at dramatically different rates.

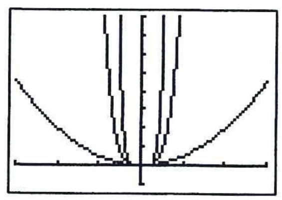
2.



[−1.5, 1.5] by [−0.5, 1.5]



[−5, 5] by [−5, 25]



[−15, 15] by [−50, 400]

The pairs $(0, 0)$, $(1, 1)$, and $(-1, 1)$ are common to all three graphs. The graphs are similar in that for $x \neq 0$, $f(x), g(x)$, and $h(x) > 0$. They are diffferent in that if $|x| < 1, f(x), g(x)$, and $h(x) \to 0$ at dramatically different rates, and if $|x| > 1, f(x), g(x)$, and $h(x) \to \infty$ at dramatically different rates.

Quick Review 2.2

1. $\sqrt[3]{x^2}$

2. $\sqrt{p^5}$

3. $\frac{1}{d^2}$

4. $\frac{1}{x^7}$

5. $\frac{1}{\sqrt[5]{q^4}}$

6. $\frac{1}{\sqrt{m^3}}$

7. $3x^{3/2}$

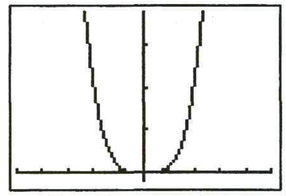
8. $2x^{5/3}$

9. $\approx 1.71x^{-4/3}$

10. $\approx 0.71x^{-1/2}$

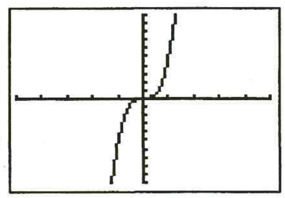
Section 2.2 Exercises

1. power $= 5$, constant $= -\frac{1}{2}$
2. power $= \frac{5}{3}$, constant $= 9$
3. not a power function
4. power $= 0$, constant $= 13$
5. power $= 1$, constant $= c^2$
6. power $= 5$, constant $= \frac{k}{2}$
7. power $= 2$, constant $= \frac{g}{2}$, indep. variable $= t$
8. power $= 3$, constant $= \frac{4\pi}{3}$, indep. variable $= r$
9. power $= -2$, constant $= k$, indep. variable $= d$
10. power $= 1$, constant $= m$
11. degree $= 0$, coefficient $= -4$
12. not a monomial function; negative exponent
13. degree $= 7$, coefficient $= -6$
14. not a monomial function; variable in exponent
15. degree $= 2$, coefficient $= 4\pi$, indep. variable $= r$
16. degree $= 1$, coefficient $= l$, indep. variable $= w$
17. $A = ks^2$
18. $V = kr^2$
19. $I = V/R$
20. $V = kT$
21. $E = mc^2$
22. $p = \sqrt{2gd}$
23. The weight w of an object varies directly with its mass m, with the constant of variation g.
24. The circumference C of a circle is proportional to its diameter D, with the constant of variation π.
25. The refractive index n of a medium is inversely proportional to v, the velocity of light in the medium, with constant of variation c, the constant velocity of light in free space.
26. The distance d traveled by a free-falling object dropped from rest varies directly with the square of its speed p, with the constant of variation $\frac{1}{2g}$.
27. $y = \frac{8}{x^2}$, power $= -2$, constant $= 8$
28. $y = -2\sqrt{x}$, power $= \frac{1}{2}$, constant $= -2$
29. (g)
30. (a)
31. (d)
32. (g)
33. (h)
34. (d)
35. Start with $y = x^4$ and shrink vertically by $\frac{2}{3}$. Since $f(-x) = \frac{2}{3}(-x)^4 = \frac{2}{3}x^4$, f is even.



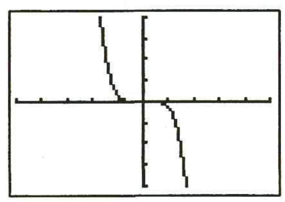
$[-5, 5]$ by $[-1, 19]$

36. Start with $y = x^3$ and stretch vertically by 5. Since $f(-x) = 5(-x)^3 = -5x^3 = -f(x)$, f is odd.



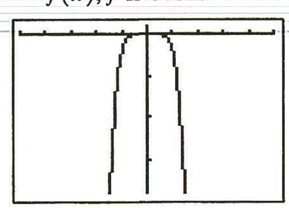
$[-5, 5]$ by $[-20, 20]$

37. Start with $y = x^5$, then stretch vertically by 1.5 and rotate across the x-axis. Since $f(-x) = -1.5(-x)^5 = 1.5x^5$ $= -f(x)$, f is odd.



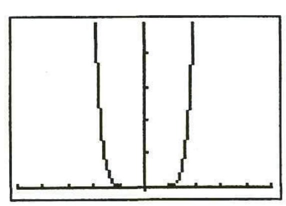
$[-5, 5]$ by $[-20, 20]$

38. Start with $y = x^6$, then stretch vertically by 2 and rotate over the x-axis. Since $f(-x) = -2(-x)^6 = -2x^6$ $= f(x)$, f is even.



$[-5, 5]$ by $[-19, 1]$

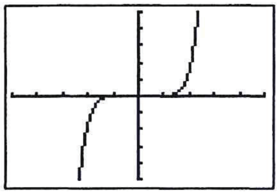
39. Start with $y = x^8$, then shrink vertically by $\frac{1}{4}$. Since $f(-x) = \frac{1}{4}(-x)^8 = \frac{1}{4}x^8 = f(x)$, f is even.



$[-5, 5]$ by $[-1, 49]$

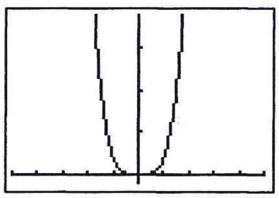
40. Start with $y = x^7$, then shrink vertically by $\frac{1}{8}$. Since

$f(-x) = \frac{1}{8}(-x)^7 = -\frac{1}{8}x^7 = -f(x)$, f is odd.



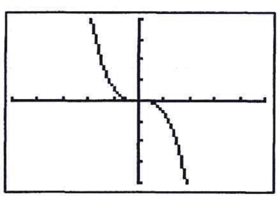
$[-5, 5]$ by $[-50, 50]$

41. power = 4, constant = 2
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Continuous
Decreasing on $(-\infty, 0)$. Increasing on $(0, \infty)$.
Even. Symmetric with respect to y-axis.
Bounded below, but not above
Local minimum at $x = 0$.
End Behavior: $\lim_{x\to-\infty} 2x^4 = \infty$, $\lim_{x\to\infty} 2x^4 = \infty$



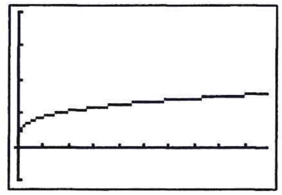
$[-5, 5]$ by $[-1, 49]$

42. power = 3, constant = −3
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Continuous
Decreasing for all x
Odd. Symmetric with respect to origin
Not bounded above or below
No local extrema
End Behavior: $\lim_{x\to-\infty} -3x^3 = \infty$, $\lim_{x\to\infty} -3x^3 = -\infty$



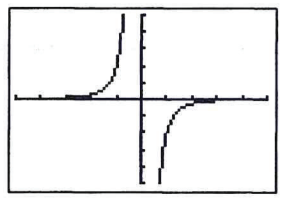
$[-5, 5]$ by $[-20, 20]$

43. power = $\frac{1}{4}$, constant = $\frac{1}{2}$
Domain: $[0, \infty)$
Range: $[0, \infty)$
Continuous
Increasing for all x
Bounded below
Neither even nor odd
Local minimum at $(0, 0)$
End Behavior: $\lim_{x\to\infty} \frac{1}{2}\sqrt[4]{x} = \infty$



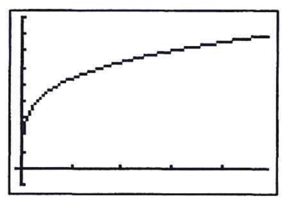
$[-1, 99]$ by $[-1, 4]$

44. power = −3, constant = −2
Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
Discontinuous at $x = 0$
Increasing on $(-\infty, 0)$. Increasing on $(0, \infty)$.
Odd. Symmetric with respect to origin
Not bounded above or below
No local extrema
Asymptote at $x = 0$.
End Behavior: $\lim_{x\to-\infty} -2x^{-3} = 0$, $\lim_{x\to\infty} -2x^{-3} = 0$.



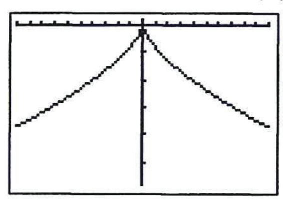
$[-5, 5]$ by $[-5, 5]$

45. $k = 3, a = \frac{1}{4}$. In the first quadrant, the function is increasing and concave down. f is undefined for $x < 0$.



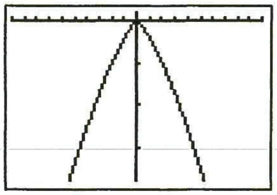
$[-1, 99]$ by $[-1, 10]$

46. $k = -4, a = \frac{2}{3}$. In the fourth quadrant, the function is decreasing and concave up. $f(-x) = -4(\sqrt[3]{(-x)^2})$
$= -4\sqrt[3]{x^2} = -4x^{\frac{2}{3}} = f(x)$, so f is even.



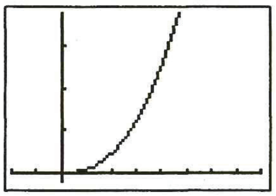
$[-10, 10]$ by $[-29, 1]$

47. $k = -2, a = \frac{4}{3}$. In the fourth quadrant, f is decreasing and concave down. $f(-x) = -2(\sqrt[3]{(-x)^4})$
$-2\left(\sqrt[3]{x^4}\right) = -2x^{\frac{4}{3}} = f(x)$, so f is even.



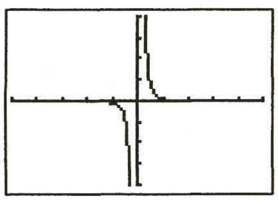
$[-10, 10]$ by $[-29, 1]$

48. $k = \frac{2}{5}, a = \frac{5}{2}$. In the first quadrant, f is increasing and concave up. f is undefined for $x < 0$.



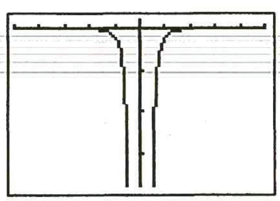
$[-2, 8]$ by $[-1, 19]$

49. $k = \frac{1}{2}, a = -3$. In the first quadrant, f is decreasing and concave up. $f(-x) = \frac{1}{2}(-x)^{-3} = \frac{1}{2(-x)^3} = -\frac{1}{2}x^{-3}$ $= -f(x)$, so f is odd.



$[-5, 5]$ by $[-20, 20]$

50. $k = -1, a = -4$. In the fourth quadrant, f is increasing and concave down. $f(-x) = -(-x)^{-4} = -\frac{1}{(-x)^4}$ $= -\frac{1}{x^4} = -x^{-4} = f(x)$, so f is even.



$[-5, 5]$ by $[-19, 1]$

51. $V = \frac{kT}{P}$, so $k = \frac{PV}{T} = \frac{(0.926 \text{ atm})(3.46 \text{ L})}{302°\text{K}}$ $= 0.0106 \frac{\text{atm–L}}{\text{K}}$

At $P = 1.452$ atm, $V = \frac{\left(\frac{0.0106 \text{ atm–L}}{\text{K}}\right)(302°\text{K})}{1.452 \text{ atm}}$ $= 2.21$ L

52. $V = kPT$, so $k = \frac{V}{PT} = \frac{(3.46 \text{ L})}{(0.926 \text{ atm})(302°\text{K})}$ $= 0.0124 \frac{\text{L}}{\text{atm–K}}$

At $T = 338°\text{K}$, $V = \left(0.0124 \frac{\text{L}}{\text{atm–K}}\right)(0.926 \text{ atm})$ $(338°\text{K}) = 3.87$ L

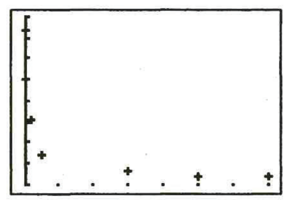
53. $n = \frac{c}{v}$, so $v = \frac{c}{n} = \frac{\left(\frac{3.00 \times 10^8 \text{ m}}{\text{sec}}\right)}{2.42} = 1.24 \times 10^8 \frac{\text{m}}{\text{sec}}$

54. $P = kv^3$, so $k = \frac{P}{v^3} = \frac{15 \text{ w}}{(10 \text{ mph})^3} = 1.5 \times 10^{-2}$

Wind Speed (mph)	Power (W)
10	15
20	120
40	960
80	7680

Since $P = kv^3$ is a cubic, power will increase significantly with only a small increase in wind speed.

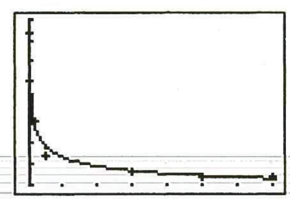
55. (a)



$[-2, 71]$ by $[50, 450]$

(b) $r \approx 231.204 \cdot w^{-0.297}$

(c)

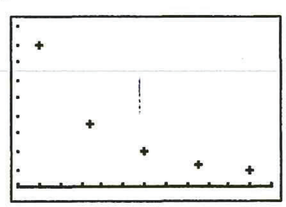


$[-2, 71]$ by $[50, 450]$

(d) Approximately 37.67 beats/min, which is very close to Clark's observed value.

56. Given that n is an integer, $n \geq 1$:
If n is odd, then $f(-x) = (-x)^n = -(x^n) = -f(x)$ and so $f(x)$ is odd.
If n is even, then $f(-x) = (-x)^n = x^n = f(x)$ and so $f(x)$ is even.

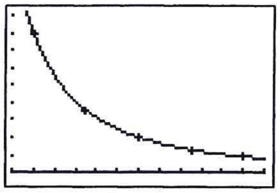
57. (a)



$[0.8, 3.2]$ by $[-0.3, 9.2]$

(b) $y \approx 7.932 \cdot x^{-1.987}$; Yes

(c)



$[0.8, 3.2]$ by $[-0.3, 9.2]$

(d) Approximately $2.76\ \frac{\text{W}}{\text{m}^2}$ and $0.697\ \frac{\text{W}}{\text{m}^2}$, respectively.

58. True. $f(-x) = (-x)^{-2/3} = [(-x)^2]^{-1/3}$
$= (x^2)^{-1/3} = x^{-2/3} = f(x)$

59. False. $f(-x) = (-x)^{1/3} = -(x^{1/3}) = -f(x)$ and so the function is odd. It is symmetric about the origin, not the y-axis.

60. $f(4) = 2(4)^{-1/2} = \frac{2}{4^{1/2}} = \frac{2}{\sqrt{4}} = \frac{2}{2} = 1.$

The answer is (a).

61. $f(0) = -3(0)^{-1/3} = -3 \cdot \frac{1}{0^{1/3}} = -3 \cdot \frac{1}{0}$ is undefined.

Also, $f(-1) = -3(-1)^{-1/3} = -3(-1) = 3$,
$f(1) = -3(1)^{-1/3} = -3(1) = -3$, and
$f(3) = -3(3)^{-1/3} \approx -2.08$. The answer is (e).

62. $f(-x) = (-x)^{2/3} = [(-x)^2]^{1/3} = (x^2)^{1/3} = x^{2/3} = f(x)$
The function is even. The answer is (b).

63. $f(x) = x^{3/2} = (x^{1/2})^3 = (\sqrt{x})^3$ is defined for $x \geq 0$.
The answer is (b).

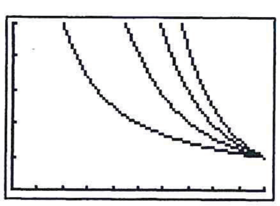
64. Answers will vary. In general, however, students will find

n even: $f(x) = k \cdot x^{\frac{m}{n}} = k \cdot \sqrt[n]{x^m}$, so f is undefined for $x < 0$.

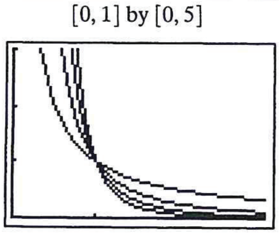
m even, n odd: $f(x) = k \cdot x^{\frac{m}{n}} = k \cdot \sqrt[n]{x^m}$; $f(-x)$
$= k \cdot \sqrt[n]{(-x)^m} = k \cdot \sqrt[n]{x^m} = f(x)$,
so f is even.

m odd, n odd: $f(x) = k \cdot x^{\frac{m}{n}} = k \cdot \sqrt[n]{x^m}$; $f(-x)$
$= k \cdot \sqrt[n]{(-x)^m} = -k \cdot \sqrt[n]{x^m}$
$= -k \cdot x^{\frac{m}{n}} = -f(x)$, so f is odd.

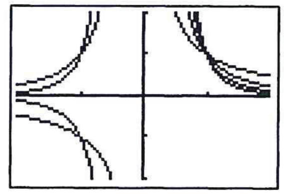
65. (a)



$[0, 1]$ by $[0, 5]$



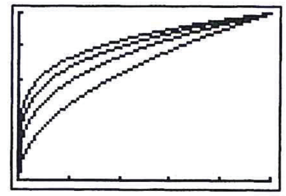
$[0, 3]$ by $[0, 3]$



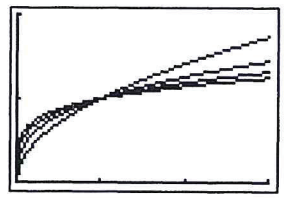
$[-2, 2]$ by $[-2, 2]$

The pair $(1, 1)$ is common to all four functions. The functions are similar in that each has an asymptote at $x = 0$ and as $x \to \pm\infty$, $f(x)$, $g(x)$, $h(x)$ and $j(x) \to 0$. They are different in that when $x < 0$, $f(x)$ and $h(x) < 0$, while $g(x)$ and $j(x) > 0$. Also, as $x \to \pm\infty$, the functions $\to 0$ at dramatically different rates.

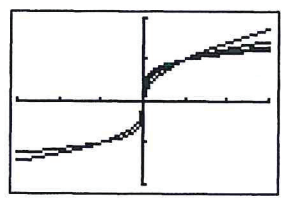
(b)



$[0, 1]$ by $[0, 1]$



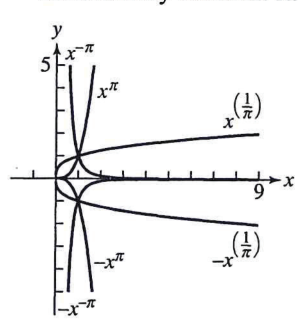
$[0, 3]$ by $[0, 2]$



$[-3, 3]$ by $[-2, 2]$

The pairs $(0, 0)$, $(1, 1)$ are common to all 4 functions. The graphs are alike in that when $x \to \infty$, $f(x)$, $g(x)$, $h(x)$, and $j(x) \to \infty$. The graphs are different, however, in that $f(x)$ and $h(x)$ do not exist (in the real plane) for $x < 0$. Also, as $x \to \infty$, $f(x)$, $g(x)$, $h(x)$, and $j(x) \to \infty$ at dramatically different rates, while as $x \to -\infty$, $g(x)$ and $j(x) \to -\infty$ at dramatically different rates.

66.



x^{π}: Since $\pi > 1$, x^{π} is increasing and concave up.

$x^{\frac{1}{\pi}}$: Since $\frac{1}{\pi} < 1$, $x^{\frac{1}{\pi}}$ is increasing and concave down.

$x^{-\pi}$: Since $x^{-\pi} = \frac{1}{x^{\pi}}$ and $\pi > 1$, $x^{-\pi}$ is decreasing and concave up.

$-x^{\pi}$: Since $\pi > 1$, $-x^{\pi}$ is decreasing and concave down.

$-x^{\frac{1}{\pi}}$: Since $\frac{1}{\pi} < 1$, $-x^{\frac{1}{\pi}}$ is decreasing and concave up.

$-x^{-\pi}$: Since $-x^{-\pi} = -\frac{1}{x^{\pi}}$ and $\pi > 1$, $-x^{-\pi}$ is increasing and concave down.

67. Our new table looks like:

Table 2.10 (revised) Average Distances and Orbit Periods for the Six Innermost Planets

Planet	Average Distance from Sun (Au)	Period of Orbit (yrs)
Mercury	0.39	0.24
Venus	0.72	0.62
Earth	1	1
Mars	1.52	1.88
Jupiter	5.20	11.86
Saturn	9.54	29.46

Source: Shupe, Dorr, Payne, Hunsiker, et al., National Geographic Atlas of the World (rev. 6th ed.). Washington, DC: National Geographic Society, 1992, plate 116.

Using this new data, we find a power function model of: $y \approx 0.99995 \cdot x^{1.50115} \approx x^{1.5}$. Since y represents years, we set $y = T$ and since x represents distance, we set $x = a$ then, $y = x^{1.5} \rightarrow T = a^{3/2} \rightarrow (T)^2 = (a^{3/2})^2 \rightarrow T^2 = a^3$.

68. Using the free-fall equations from Section 2.1, we know that $s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$. Therefore $d = s_0 - s$ $= s_0 - \left(-\frac{1}{2}gt^2 + v_0t + s_0\right) = \frac{1}{2}gt^2 - v_0t$.

In this case, the initial velocity was zero, so $v_0 = 0$ and $d = \frac{1}{2}gt^2 - 0 \cdot t = \frac{1}{2}gt^2$. Solving for t we have $t = \sqrt{\frac{2d}{g}}$.

Then, $p = gt = g.\frac{\sqrt{2d}}{\sqrt{g}} = \sqrt{2gd}$. Yes.

69. If f is even, $f(x) = f(-x)$, so $\frac{1}{f(x)} = \frac{1}{f(-x)}$ $(f(x) \neq 0)$. Since $g(x) = \frac{1}{f(x)} = \frac{1}{f(-x)} = g(-x)$, g is also even. If g is even, $g(x) = g(-x)$, so $g(-x) = \frac{1}{f(-x)}$ $= g(x) = \frac{1}{f(x)}$. Since $\frac{1}{f(-x)} = \frac{1}{f(x)}$, $f(-x) = f(x)$, and f is even. If f is odd, $f(x) = -f(x)$, so $\frac{1}{f(x)} = -\frac{1}{f(x)}$, $f(x) \neq 0$. Since $g(x) = \frac{1}{f(x)} = -\frac{1}{f(x)}$ $= -g(x)$, g is also odd. If g is odd, $g(x) = g(-x)$, so $g(-x) = \frac{1}{f(-x)} = -g(x) = -\frac{1}{f(x)}$. Since $\frac{1}{f(-x)} = -\frac{1}{f(x)}$, $f(-x) = -f(x)$, and f is odd.

70. $f(x)$ is even if and only if $\frac{1}{f(x)}$ is also even. Using this result, $f(x) = k \cdot x^a$ is even if and only if $\frac{1}{f(x)} = \frac{1}{k \cdot x^a}$ $= \frac{1}{k}x^{-a} = k_2x^{-a} = g(x)$ is even $(f(x) \neq 0)$, $f(x)$ is odd if and only if $\frac{1}{f(x)}$ is also odd. Using this result, $f(x) = k \cdot x^a$ is odd if and only if $\frac{1}{f(x)} = \frac{1}{k \cdot x^a} = \frac{1}{k}x^{-a}$ $= k_3\,x^{-a} = g(x)$ is odd $(f(x) \neq 0)$.

71. **(a)** The force F acting on an object varies jointly as the mass m of the object and the acceleration a of the object.

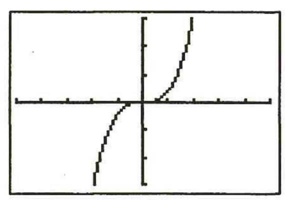
(b) The kinetic energy KE of an object varies jointly as the mass m of the object and square of the velocity v of the object.

(c) The force of gravity F acting on two objects varies jointly as the product m_1m_2 of the objects' masses and the inverse of the distance r between their centers, with the constant of variation G, the universal gravitational constant.

Section 2.3 Polynomial Functions of Higher Degree with Modeling

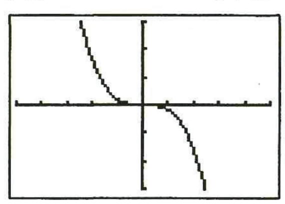
Exploration 1

1. **(a)** $\lim_{x\to\infty} 2x^3 = \infty$, $\lim_{x\to-\infty} 2x^3 = -\infty$



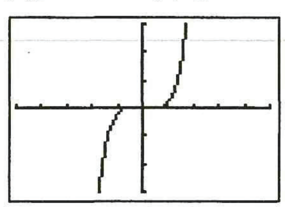
[−5, 5] by [−15, 15]

(b) $\lim_{x\to\infty} (-x^3) = -\infty$, $\lim_{x\to-\infty} (-x^3) = \infty$



[−5, 5] by [−15, 15]

(c) $\lim_{x\to\infty} x^5 = \infty$, $\lim_{x\to-\infty} x^5 = -\infty$



[−5, 5] by [−15, 15]