$x^{-\pi}$: Since $x^{-\pi} = \frac{1}{x^{\pi}}$ and $\pi > 1$, $x^{-\pi}$ is decreasing and concave up.

$-x^{\pi}$: Since $\pi > 1$, $-x^{\pi}$ is decreasing and concave down.

$-x^{\frac{1}{\pi}}$: Since $\frac{1}{\pi} < 1$, $-x^{\frac{1}{\pi}}$ is decreasing and concave up.

$-x^{-\pi}$: Since $-x^{-\pi} = -\frac{1}{x^{\pi}}$ and $\pi > 1$, $-x^{-\pi}$ is increasing and concave down.

67. Our new table looks like:

Table 2.10 (revised) Average Distances and Orbit Periods for the Six Innermost Planets

Planet	Average Distance from Sun (Au)	Period of Orbit (yrs)
Mercury	0.39	0.24
Venus	0.72	0.62
Earth	1	1
Mars	1.52	1.88
Jupiter	5.20	11.86
Saturn	9.54	29.46

Source: Shupe, Dorr, Payne, Hunsiker, et al., National Geographic Atlas of the World (rev. 6th ed.). Washington, DC: National Geographic Society, 1992, plate 116.

Using this new data, we find a power function model of: $y \approx 0.99995 \cdot x^{1.50115} \approx x^{1.5}$. Since y represents years, we set $y = T$ and since x represents distance, we set $x = a$ then, $y = x^{1.5} \rightarrow T = a^{3/2} \rightarrow (T)^2 = (a^{3/2})^2 \rightarrow T^2 = a^3$.

68. Using the free-fall equations from Section 2.1, we know that $s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$. Therefore $d = s_0 - s$

$= s_0 - \left(-\frac{1}{2}gt^2 + v_0t + s_0\right) = \frac{1}{2}gt^2 - v_0t.$

In this case, the initial velocity was zero, so $v_0 = 0$ and

$d = \frac{1}{2}gt^2 - 0 \cdot t = \frac{1}{2}gt^2$. Solving for t we have $t = \sqrt{\frac{2d}{g}}$.

Then, $p = gt = g.\frac{\sqrt{2d}}{\sqrt{g}} = \sqrt{2gd}$. Yes.

69. If f is even, $f(x) = f(-x)$, so $\frac{1}{f(x)} = \frac{1}{f(-x)}$ $(f(x) \neq 0)$. Since $g(x) = \frac{1}{f(x)} = \frac{1}{f(-x)} = g(-x)$, g is also even. If g is even, $g(x) = g(-x)$, so $g(-x) = \frac{1}{f(-x)}$ $= g(x) = \frac{1}{f(x)}$. Since $\frac{1}{f(-x)} = \frac{1}{f(x)}$, $f(-x) = f(x)$, and f is even. If f is odd, $f(x) = -f(x)$, so $\frac{1}{f(x)} = -\frac{1}{f(x)}$, $f(x) \neq 0$. Since $g(x) = \frac{1}{f(x)} = -\frac{1}{f(x)}$ $= -g(x)$, g is also odd. If g is odd, $g(x) = g(-x)$, so $g(-x) = \frac{1}{f(-x)} = -g(x) = -\frac{1}{f(x)}$. Since $\frac{1}{f(-x)} = -\frac{1}{f(x)}$, $f(-x) = -f(x)$, and f is odd.

70. $f(x)$ is even if and only if $\frac{1}{f(x)}$ is also even. Using this result, $f(x) = k \cdot x^a$ is even if and only if $\frac{1}{f(x)} = \frac{1}{k \cdot x^a}$ $= \frac{1}{k}x^{-a} = k_2x^{-a} = g(x)$ is even $(f(x) \neq 0)$, $f(x)$ is odd if and only if $\frac{1}{f(x)}$ is also odd. Using this result, $f(x) = k \cdot x^a$ is odd if and only if $\frac{1}{f(x)} = \frac{1}{k \cdot x^a} = \frac{1}{k}x^{-a}$ $= k_3\, x^{-a} = g(x)$ is odd $(f(x) \neq 0)$.

71. **(a)** The force F acting on an object varies jointly as the mass m of the object and the acceleration a of the object.

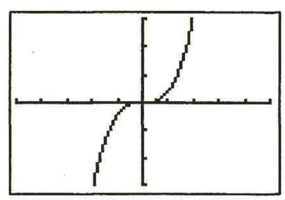
(b) The kinetic energy KE of an object varies jointly as the mass m of the object and square of the velocity v of the object.

(c) The force of gravity F acting on two objects varies jointly as the product m_1m_2 of the objects' masses and the inverse of the distance r between their centers, with the constant of variation G, the universal gravitational constant.

Section 2.3 Polynomial Functions of Higher Degree with Modeling

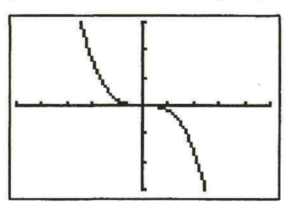
Exploration 1

1. **(a)** $\lim_{x\to\infty} 2x^3 = \infty$, $\lim_{x\to-\infty} 2x^3 = -\infty$



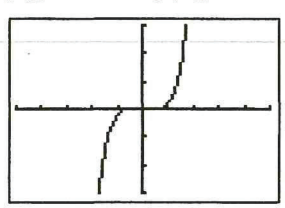
$[-5, 5]$ by $[-15, 15]$

(b) $\lim_{x\to\infty} (-x^3) = -\infty$, $\lim_{x\to-\infty} (-x^3) = \infty$



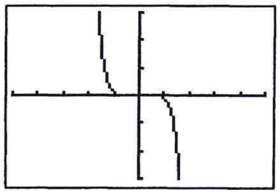
$[-5, 5]$ by $[-15, 15]$

(c) $\lim_{x\to\infty} x^5 = \infty$, $\lim_{x\to-\infty} x^5 = -\infty$



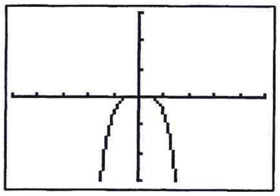
$[-5, 5]$ by $[-15, 15]$

(d) $\lim_{x\to\infty} (-0.5x^7) = -\infty,\ \lim_{x\to-\infty} (-0.5x^7) = \infty$



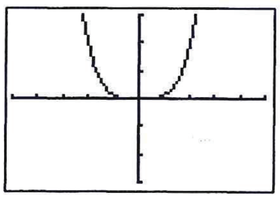
[−5, 5] by [−15, 15]

2. (a) $\lim_{x\to\infty} (-3x^4) = -\infty,\ \lim_{x\to-\infty} (-3x^4) = -\infty$



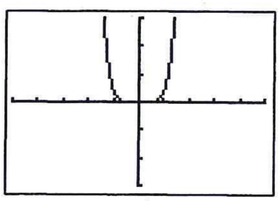
[−5, 5] by [−15, 15]

(b) $\lim_{x\to\infty} 0.6x^4 = \infty,\ \lim_{x\to-\infty} 0.6x^4 = \infty$



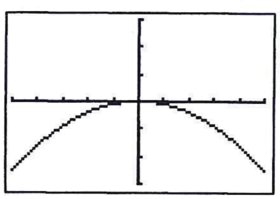
[−5, 5] by [−15, 15]

(c) $\lim_{x\to\infty} 2x^6 = \infty,\ \lim_{x\to-\infty} 2x^6 = \infty$



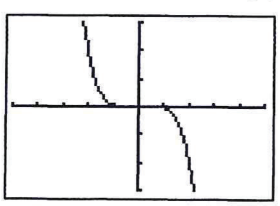
[−5, 5] by [−15, 15]

(d) $\lim_{x\to\infty} (-0.5x^2) = -\infty,\ \lim_{x\to-\infty} (-0.5x^2) = -\infty$



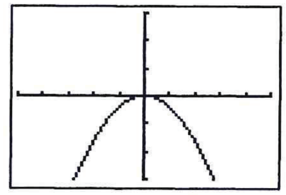
[−5, 5] by [−15, 15]

3. (a) $\lim_{x\to\infty} (-0.3x^5) = -\infty,\ \lim_{x\to-\infty} (-0.3x^5) = \infty$



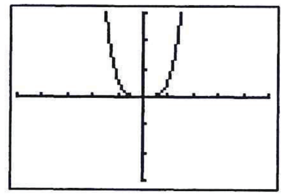
[−5, 5] by [−15, 15]

(b) $\lim_{x\to\infty} (-2x^2) = -\infty,\ \lim_{x\to-\infty} (-2x^2) = -\infty$



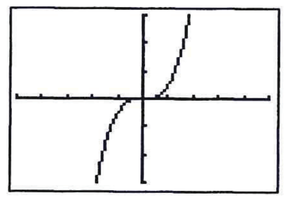
[−5, 5] by [−15, 15]

(c) $\lim_{x\to\infty} 3x^4 = \infty,\ \lim_{x\to-\infty} 3x^4 = \infty$



[−5, 5] by [−15, 15]

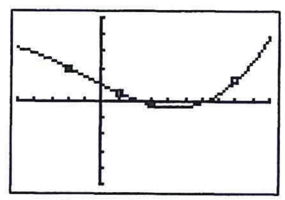
(d) $\lim_{x\to\infty} 2.5x^3 = \infty,\ \lim_{x\to-\infty} 2.5x^3 = -\infty$



[−5, 5] by [−15, 15]

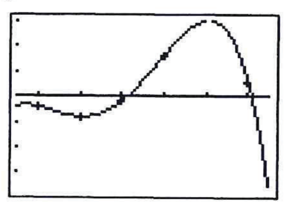
Exploration 2

1. $y = 0.0061x^3 + 0.0177x^2 - 0.5007x + 0.9769$ It is an exact fit, which we expect with only 4 data points!



[−5, 10] by [−5, 5]

2. $y = -0.375x^4 + 6.917x^3 - 44.125x^2 + 116.583x - 111$ It is an exact fit, exactly what we expect with only 5 data points!



[2.5, 8.5] by [−18, 15]

Quick Review 2.3

1. $(x - 4)(x + 3)$

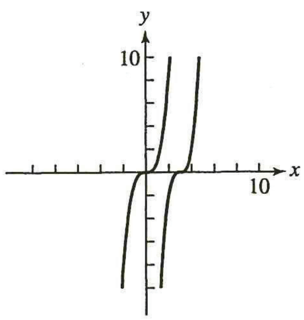
2. $(x - 7)(x - 4)$

3. $(3x - 2)(x - 3)$

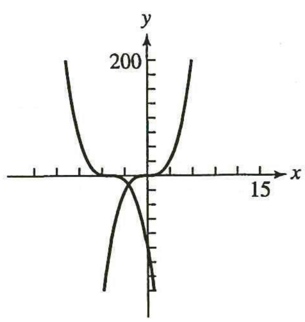
4. $(2x - 1)(3x - 1)$
5. $x(3x - 2)(x - 1)$
6. $2x(3x - 2)(x - 3)$
7. $x = 0, x = 1$
8. $x = 0, x = -2, x = 5$
9. $x = -6, x = -3, x = 1.5$
10. $x = -6, x = -4, x = 5$

Section 2.3 Exercises

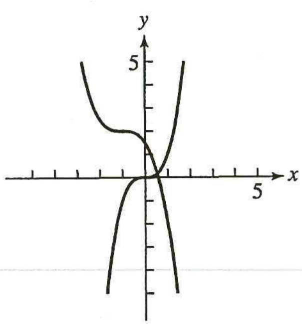
1. Start with $y = x^3$, shift to the right by 3 units, and then stretch vertically by 2. y-intercept: $(0, -54)$



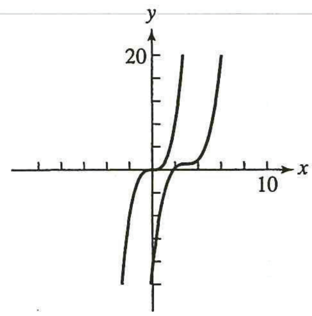
2. Start with $y = x^3$, shift to the left by 5 units, and then reflect over the x-axis. y-intercept: $(0, -125)$



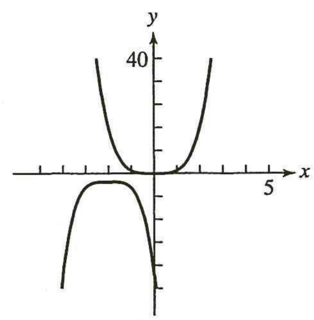
3. Start with $y = x^3$, shift to the left by 1 unit, vertically shrink by $\frac{1}{2}$, reflect over the x-axis, and then vertically shift up 2 units. y-intercept: $\left(0, \frac{3}{2}\right)$



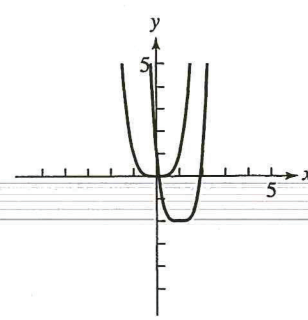
4. Start with $y = x^3$, shift to the right by 3 units, vertically shrink by $\frac{2}{3}$, and vertically shift up one unit. y-intercept: $(0, -17)$



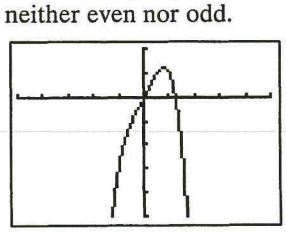
5. Start with $y = x^4$, shift to the left 2 units, vertically stretch by 2, reflect over the x-axis, and vertically shift down 3 units. y-intercept: $(0, -35)$



6. Start with $y = x^4$, shift to the right 1 unit, vertically stretch by 3, and vertically shift down 2 units. y-intercept: $(0, 1)$

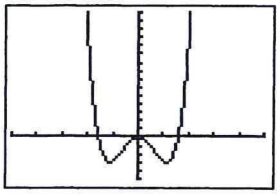


7. local maximum: $\approx (0.79, 1.19)$, zeros: $x = 0$ and $x \approx 1.26$. The general shape of f is like $y = -x^4$, but near the origin, f behaves a lot like its other term, $2x$. f is neither even nor odd.



$[-5, 5]$ by $[-5, 2]$

8. local max at $(0, 0)$ and local minima at $(1.12, -3.13)$ and $(-1.12, -3.13)$, zeros: $x = 0$, $x \approx 1.58$, $x \approx -1.58$. f behaves a lot like $y = 2x^4$ except in the interval $[-1.58, 1.58]$, where it behaves more like its second building block term, $-5x^2$.



$[-5, 5]$ by $[-5, 15]$

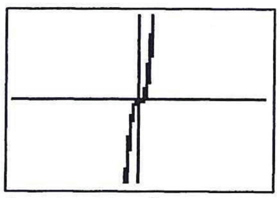
9. Cubic function, positive leading coefficient. The answer is (c).

10. Cubic function, negative leading coefficient. The answer is (b).

11. Higher than cubic, positive leading coefficient. The answer is (a).

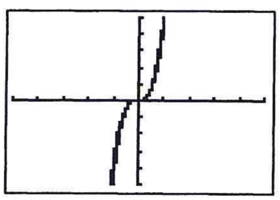
12. Higher than cubic, negative leading coefficient. The answer is (d).

13. One possibility:



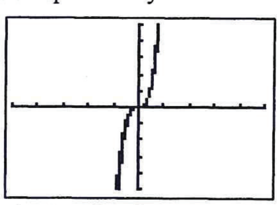
$[-100, 100]$ by $[-1000, 1000]$

14. One possibility:



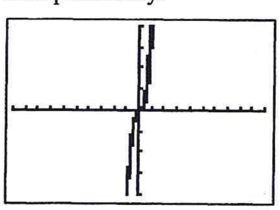
$[-50, 50]$ by $[-1000, 1000]$

15. One possibility:



$[-50, 50]$ by $[-1000, 1000]$

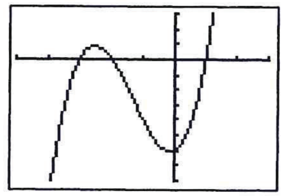
16. One possibility:



$[-100, 100]$ by $[-2000, 2000]$

For #17–24, when one end of a polynomial function's graph curves up into Quadrant I or II, this indicates a limit at ∞. And when an end curves down into Quadrant III or IV, this indicates a limit at $-\infty$.

17.

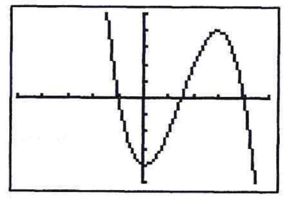


$[-5, 3]$ by $[-8, 3]$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = -\infty$

18.

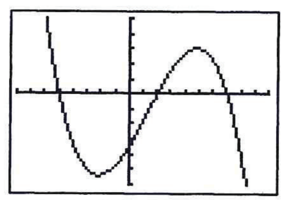


$[-5, 5]$ by $[-15, 15]$

$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = \infty$

19.

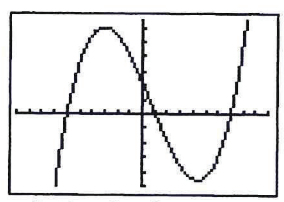


$[-8, 10]$ by $[-120, 100]$

$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = \infty$

20.

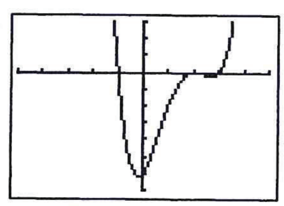


$[-10, 10]$ by $[-100, 130]$

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = -\infty$

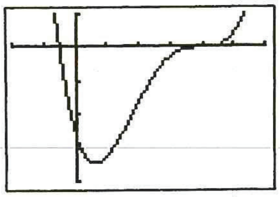
21.



$[-5, 5]$ by $[-14, 6]$

$\lim_{x \to \infty} f(x) = \infty$

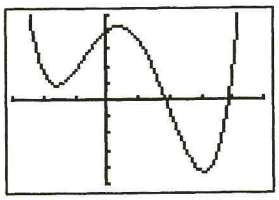
$\lim_{x \to -\infty} f(x) = \infty$

22. 

[−2, 6] by [−100, 25]

$\lim_{x \to \infty} f(x) = \infty$

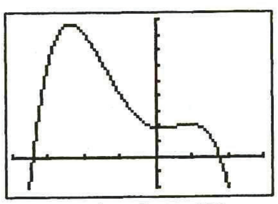
$\lim_{x \to -\infty} f(x) = \infty$

23. 

[−3, 5] by [−50, 50]

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = \infty$

24. 

[−4, 3] by [−20, 90]

$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = -\infty$

For #25–28, the end behavior of a polynomial is governed by the highest-degree term.

25. $\lim_{x \to \infty} f(x) = \infty$, $\lim_{x \to -\infty} f(x) = \infty$

26. $\lim_{x \to \infty} f(x) = -\infty$, $\lim_{x \to -\infty} f(x) = \infty$

27. $\lim_{x \to \infty} f(x) = -\infty$, $\lim_{x \to -\infty} f(x) = \infty$

28. $\lim_{x \to \infty} f(x) = -\infty$, $\lim_{x \to -\infty} f(x) = -\infty$

29. (a); There are 3 zeros: they are −2.5, 1, and 1.1.

30. (b); There are 3 zeros: they are 0.4, approximately 0.429 (actually 3/7), and 3.

31. (c); There are 3 zeros: approximately −0.273 (actually −3/11), −0.25, and 1.

32. (d); There are 3 zeros: −2, 0.5, and 3.

For #33–35, factor or apply the quadratic formula.

33. −4 and 2

34. −2 and 2/3

35. 2/3 and −1/3

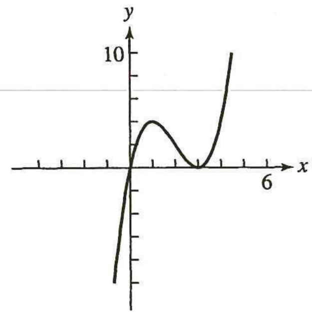
For #36–38, factor out x, then factor or apply the quadratic formula.

36. 0, −5, and 5

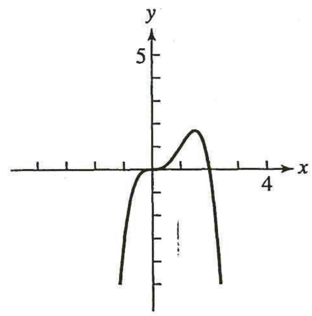
37. 0, −2/3, and 1

38. 0, −1, and 2

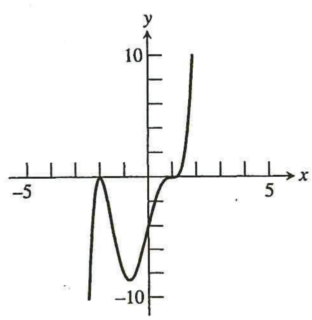
39. degree: 3, zeros: $x = 0$ (one, crosses x-axis), $x = 3$ (two, does not cross x-axis)



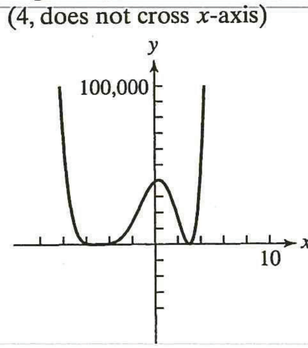
40. degree: 4, zeros: $x = 0$ (3, crosses x-axis), $x = 2$ (one, crosses x-axis)



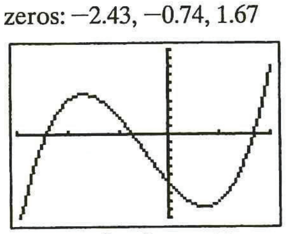
41. degree: 5, zeros: $x = 1$ (3, crosses x-axis), $x = -2$ (two, does not cross x-axis)



42. degree: 6, zeros: $x = 3$ (2, does not cross x-axis), $x = -5$ (4, does not cross x-axis)



43. zeros: −2.43, −0.74, 1.67



[−3, 2] by [−10, 10]