Vocabulary and Core Concept Check

1. **WRITING** Describe how to multiply and divide two rational expressions.

2. **WHICH ONE DOESN'T BELONG?** Which rational expression does *not* belong with the other three? Explain your reasoning.

$$\dfrac{x-4}{x^2} \qquad \dfrac{x^2+4x-12}{x^2+6x} \qquad \dfrac{9+x}{3x^2} \qquad \dfrac{x^2-x-12}{x^2-6x}$$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, simplify the expression, if possible. *(See Example 1.)*

3. $\dfrac{2x^2}{3x^2-4x}$

4. $\dfrac{7x^3-x^2}{2x^3}$

5. $\dfrac{x^2-3x-18}{x^2-7x+6}$

6. $\dfrac{x^2+13x+36}{x^2-7x+10}$

7. $\dfrac{x^2+11x+18}{x^3+8}$

8. $\dfrac{x^2-7x+12}{x^3-27}$

9. $\dfrac{32x^4-50}{4x^3-12x^2-5x+15}$

10. $\dfrac{3x^3-3x^2+7x-7}{27x^4-147}$

In Exercises 11–20, find the product. *(See Examples 2, 3, and 4.)*

11. $\dfrac{4xy^3}{x^2y} \cdot \dfrac{y}{8x}$

12. $\dfrac{48x^5y^3}{y^4} \cdot \dfrac{x^2y}{6x^3y^2}$

13. $\dfrac{x^2(x-4)}{x-3} \cdot \dfrac{(x-3)(x+6)}{x^3}$

14. $\dfrac{x^3(x+5)}{x-9} \cdot \dfrac{(x-9)(x+8)}{3x^3}$

15. $\dfrac{x^2-3x}{x-2} \cdot \dfrac{x^2+x-6}{x}$

16. $\dfrac{x^2-4x}{x-1} \cdot \dfrac{x^2+3x-4}{2x}$

17. $\dfrac{x^2+3x-4}{x^2+4x+4} \cdot \dfrac{2x^2+4x}{x^2-4x+3}$

18. $\dfrac{x^2-x-6}{4x^3} \cdot \dfrac{2x^2+2x}{x^2+5x+6}$

19. $\dfrac{x^2+5x-36}{x^2-49} \cdot (x^2-11x+28)$

20. $\dfrac{x^2-x-12}{x^2-16} \cdot (x^2+2x-8)$

21. **ERROR ANALYSIS** Describe and correct the error in simplifying the rational expression.

$$\boxed{\dfrac{x^2+\overset{2}{\cancel{16}}x+\overset{3}{\cancel{48}}}{x^2+\underset{1}{\cancel{8}}x+\underset{1}{\cancel{16}}} = \dfrac{x^2+2x+3}{x^2+x+1}}$$

22. **ERROR ANALYSIS** Describe and correct the error in finding the product.

$$\dfrac{x^2-25}{3-x} \cdot \dfrac{x-3}{x+5} = \dfrac{(x+5)(x-5)}{3-x} \cdot \dfrac{x-3}{x+5}$$
$$= \dfrac{(x+5)(x-5)(\cancel{x-3})}{(3-x)(x+5)}$$
$$= x-5, x \neq 3, x \neq -5$$

23. **USING STRUCTURE** Which rational expression is in simplified form?

Ⓐ $\dfrac{x^2-x-6}{x^2+3x+2}$ Ⓑ $\dfrac{x^2+6x+8}{x^2+2x-3}$

Ⓒ $\dfrac{x^2-6x+9}{x^2-2x-3}$ Ⓓ $\dfrac{x^2+3x-4}{x^2+x-2}$

24. **COMPARING METHODS** Find the product below by multiplying the numerators and denominators, then simplifying. Then find the product by simplifying each expression, then multiplying. Which method do you prefer? Explain.

$$\dfrac{4x^2y}{2x^3} \cdot \dfrac{12y^4}{24x^2}$$

25. WRITING Compare the function

$$f(x) = \frac{(3x - 7)(x + 6)}{(3x - 7)}$$ to the function $g(x) = x + 6$.

26. MODELING WITH MATHEMATICS You build a model for the construction of a new building. Write a model in terms of x for the total area of the base of the new building.

$$\frac{x^2 - 7x + 10}{6x - 12}$$
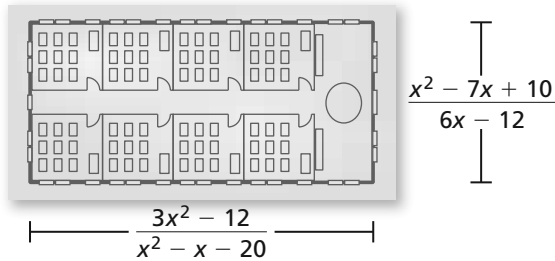
$$\frac{3x^2 - 12}{x^2 - x - 20}$$

In Exercises 27–34, find the quotient. *(See Examples 5 and 6.)*

27. $\dfrac{32x^3 y}{y^8} \div \dfrac{y^7}{8x^4}$

28. $\dfrac{2xyz}{x^3 z^3} \div \dfrac{6y^4}{2x^2 z^2}$

29. $\dfrac{x^2 - x - 6}{2x^4 - 6x^3} \div \dfrac{x + 2}{4x^3}$

30. $\dfrac{2x^2 - 12x}{x^2 - 7x + 6} \div \dfrac{2x}{3x - 3}$

31. $\dfrac{x^2 - x - 6}{x + 4} \div (x^2 - 6x + 9)$

32. $\dfrac{x^2 - 5x - 36}{x + 2} \div (x^2 - 18x + 81)$

33. $\dfrac{x^2 + 9x + 18}{x^2 + 6x + 8} \div \dfrac{x^2 - 3x - 18}{x^2 + 2x - 8}$

34. $\dfrac{x^2 - 3x - 40}{x^2 + 8x - 20} \div \dfrac{x^2 + 13x + 40}{x^2 + 12x + 20}$

35. PROBLEM SOLVING Manufacturers often package products in a way that uses the least amount of material. One measure of the efficiency of a package is the ratio of its surface area to its volume. The smaller the ratio, the more efficient the packaging.

a. Write an expression for the efficiency ratio $\dfrac{S}{V}$.

b. Find the efficiency ratio for each can listed in the table.

|	**Soup**	**Coffee**	**Paint**
Height, x	10.2 cm	15.9 cm	19.4 cm
Radius, r	3.4 cm	7.8 cm	8.4 cm

c. Rank the three cans in part (b) according to efficiency. Explain.

36. PROBLEM SOLVING A company makes a tin to hold popcorn. The tin is a rectangular prism with a square base. The company is designing a new tin with the same base and twice the height of the old tin.

a. Write an expression for the efficiency ratio $\dfrac{S}{V}$.

b. Find the efficiency ratio for each tin.

c. Did the company make a good decision by creating the new tin? Explain.
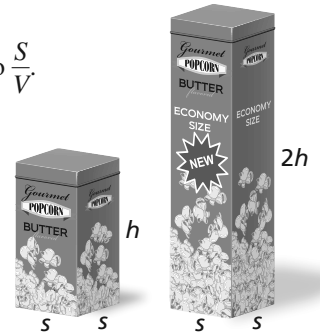
37. MODELING WITH MATHEMATICS The total amount I (in billions of dollars) of healthcare expenditures and the residential population P (in thousands) in the United States can be modeled by

$$I = \frac{171t + 1361}{1 + 0.018t} \text{ and } P = 2960t + 278{,}649$$

where t is the number of years since 2000. Find a model M for the annual healthcare expenditures per resident. Estimate the annual healthcare expenditures per resident in 2010. *(See Example 7.)*

38. MODELING WITH MATHEMATICS The total amount I (in millions of dollars) of school expenditures from prekindergarten to a college level and the enrollment P (in thousands) in prekindergarten through college in the United States can be modeled by

$$I = \frac{17{,}913t + 709{,}569}{1 - 0.028t} \text{ and } P = 590.6t + 70{,}219$$

where t is the number of years since 2001. Find a model M for the annual education expenditures per student. Estimate the annual education expenditures per student in 2009.
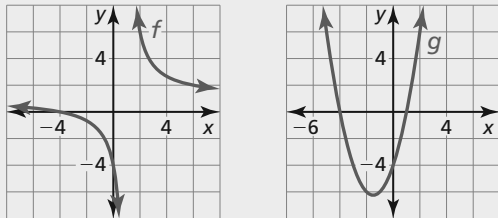
39. USING EQUATIONS Refer to the population model P in Exercise 37.

a. Interpret the meaning of the coefficient of t.

b. Interpret the meaning of the constant term.

40. HOW DO YOU SEE IT? Use the graphs of f and g to determine the excluded values of the functions $h(x) = (fg)(x)$ and $k(x) = \left(\dfrac{f}{g}\right)(x)$. Explain your reasoning.

41. DRAWING CONCLUSIONS Complete the table for the function $y = \dfrac{x+4}{x^2-16}$. Then use the *trace* feature of a graphing calculator to explain the behavior of the function at $x = -4$.
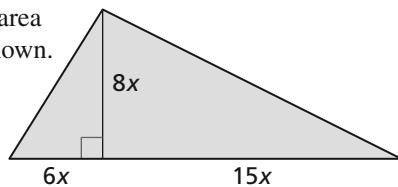
x	y
−3.5	
−3.8	
−3.9	
−4.1	
−4.2	

42. MAKING AN ARGUMENT You and your friend are asked to state the domain of the expression below.

$$\frac{x^2 + 6x - 27}{x^2 + 4x - 45}$$

Your friend claims the domain is all real numbers except 5. You claim the domain is all real numbers except −9 and 5. Who is correct? Explain.

43. MATHEMATICAL CONNECTIONS Find the ratio of the perimeter to the area of the triangle shown.

8x
6x
15x

**44. CRITICAL THINKING** Find the expression that makes the following statement true.

$$\frac{x-5}{x^2+2x-35} \div \frac{\boxed{}}{x^2-3x-10} = \frac{x+2}{x+7}$$

USING STRUCTURE In Exercises 45 and 46, perform the indicated operations.

45. $\dfrac{2x^2+x-15}{2x^2-11x-21} \cdot (6x+9) \div \dfrac{2x-5}{3x-21}$

46. $(x^3+8) \cdot \dfrac{x-2}{x^2-2x+4} \div \dfrac{x^2-4}{x-6}$

47. REASONING Animals that live in cold climates must avoid losing heat to survive. Animals with a minimum amount of surface area exposed to the environment can better conserve body heat. Penguins have a cylindrical shape.
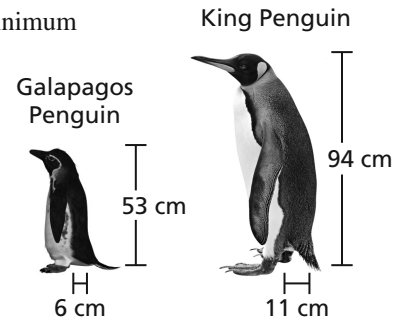
King Penguin
Galapagos Penguin
94 cm
53 cm
6 cm
11 cm
Not drawn to scale

a. Write an expression for the efficiency ratio $\dfrac{S}{V}$.

b. Find the efficiency ratio for each penguin.

c. Which penguin lives in a colder climate? Explain your reasoning.

48. THOUGHT PROVOKING Is it possible to write two radical functions whose product when graphed is a parabola and whose quotient when graphed is a hyperbola? Justify your answer.

49. REASONING Find two rational functions f and g that have the stated product and quotient.

$$(fg)(x) = x^2, \quad \left(\frac{f}{g}\right)(x) = \frac{(x-1)^2}{(x+2)^2}$$

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. *(Skills Review Handbook)*

50. $\frac{1}{2}x + 4 = \frac{3}{2}x + 5$

51. $\frac{1}{3}x - 2 = \frac{3}{4}x$

52. $\frac{1}{4}x - \frac{3}{5} = \frac{9}{2}x - \frac{4}{5}$

53. $\frac{1}{2}x + \frac{1}{3} = \frac{3}{4}x - \frac{1}{5}$

Write the prime factorization of the number. If the number is prime, then write *prime*.
(Skills Review Handbook)

54. 42

55. 91

56. 72

57. 79