7.4 Lesson

Core Vocabulary

complex fraction, *p. 387*

Previous
rational numbers
reciprocal

What You Will Learn

- Add or subtract rational expressions.
- Rewrite rational expressions and graph the related function.
- Simplify complex fractions.

Adding or Subtracting Rational Expressions

As with numerical fractions, the procedure used to add (or subtract) two rational expressions depends upon whether the expressions have like or unlike denominators. To add (or subtract) rational expressions with like denominators, simply add (or subtract) their numerators. Then place the result over the common denominator.

Core Concept

Adding or Subtracting with Like Denominators

Let a, b, and c be expressions with $c \neq 0$.

Addition

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

Subtraction

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

EXAMPLE 1 Adding or Subtracting with Like Denominators

a. $\frac{7}{4x} + \frac{3}{4x} = \frac{7+3}{4x} = \frac{10}{4x} = \frac{5}{2x}$ — Add numerators and simplify.

b. $\frac{2x}{x+6} - \frac{5}{x+6} = \frac{2x-5}{x+6}$ — Subtract numerators.

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Find the sum or difference.

1. $\frac{8}{12x} - \frac{5}{12x}$ **2.** $\frac{2}{3x^2} + \frac{1}{3x^2}$ **3.** $\frac{4x}{x-2} - \frac{x}{x-2}$ **4.** $\frac{2x^2}{x^2+1} + \frac{2}{x^2+1}$

To add (or subtract) two rational expressions with *unlike* denominators, find a common denominator. Rewrite each rational expression using the common denominator. Then add (or subtract).

Core Concept

Adding or Subtracting with Unlike Denominators

Let a, b, c, and d be expressions with $c \neq 0$ and $d \neq 0$.

Addition

$$\frac{a}{c} + \frac{b}{d} = \frac{ad}{cd} + \frac{bc}{cd} = \frac{ad+bc}{cd}$$

Subtraction

$$\frac{a}{c} - \frac{b}{d} = \frac{ad}{cd} - \frac{bc}{cd} = \frac{ad-bc}{cd}$$

You can always find a common denominator of two rational expressions by multiplying the denominators, as shown above. However, when you use the least common denominator (LCD), which is the least common multiple (LCM) of the denominators, simplifying your answer may take fewer steps.

To find the LCM of two (or more) expressions, factor the expressions completely. The LCM is the product of the highest power of each factor that appears in any of the expressions.

EXAMPLE 2 Finding a Least Common Multiple (LCM)

Find the least common multiple of $4x^2 - 16$ and $6x^2 - 24x + 24$.

SOLUTION

Step 1 Factor each polynomial. Write numerical factors as products of primes.

$$4x^2 - 16 = 4(x^2 - 4) = (2^2)(x + 2)(x - 2)$$

$$6x^2 - 24x + 24 = 6(x^2 - 4x + 4) = (2)(3)(x - 2)^2$$

Step 2 The LCM is the product of the highest power of each factor that appears in either polynomial.

$$\text{LCM} = (2^2)(3)(x + 2)(x - 2)^2 = 12(x + 2)(x - 2)^2$$

EXAMPLE 3 Adding with Unlike Denominators

Find the sum $\dfrac{7}{9x^2} + \dfrac{x}{3x^2 + 3x}$.

SOLUTION

Method 1 Use the definition for adding rational expressions with unlike denominators.

$$\frac{7}{9x^2} + \frac{x}{3x^2 + 3x} = \frac{7(3x^2 + 3x) + x(9x^2)}{9x^2(3x^2 + 3x)} \qquad \frac{a}{c} + \frac{b}{d} = \frac{ad + bc}{cd}$$

$$= \frac{21x^2 + 21x + 9x^3}{9x^2(3x^2 + 3x)} \qquad \text{Distributive Property}$$

$$= \frac{\cancel{3x}(3x^2 + 7x + 7)}{9x^2(x + 1)(\cancel{3x})} \qquad \text{Factor. Divide out common factors.}$$

$$= \frac{3x^2 + 7x + 7}{9x^2(x + 1)} \qquad \text{Simplify.}$$

Method 2 Find the LCD and then add. To find the LCD, factor each denominator and write each factor to the highest power that appears in either denominator. Note that $9x^2 = 3^2x^2$ and $3x^2 + 3x = 3x(x + 1)$, so the LCD is $9x^2(x + 1)$.

$$\frac{7}{9x^2} + \frac{x}{3x^2 + 3x} = \frac{7}{9x^2} + \frac{x}{3x(x + 1)} \qquad \text{Factor second denominator.}$$

$$= \frac{7}{9x^2} \cdot \frac{x + 1}{x + 1} + \frac{x}{3x(x + 1)} \cdot \frac{3x}{3x} \qquad \text{LCD is } 9x^2(x + 1).$$

$$= \frac{7x + 7}{9x^2(x + 1)} + \frac{3x^2}{9x^2(x + 1)} \qquad \text{Multiply.}$$

$$= \frac{3x^2 + 7x + 7}{9x^2(x + 1)} \qquad \text{Add numerators.}$$

Note in Examples 1 and 3 that when adding or subtracting rational expressions, the result is a rational expression. In general, similar to rational numbers, rational expressions are closed under addition and subtraction.

EXAMPLE 4 Subtracting with Unlike Denominators

Find the difference $\frac{x+2}{2x-2} - \frac{-2x-1}{x^2-4x+3}$.

SOLUTION

COMMON ERROR

When subtracting rational expressions, remember to distribute the negative sign to all the terms in the quantity that is being subtracted.

$$\frac{x+2}{2x-2} - \frac{-2x-1}{x^2-4x+3} = \frac{x+2}{2(x-1)} - \frac{-2x-1}{(x-1)(x-3)}$$ Factor each denominator.

$$= \frac{x+2}{2(x-1)} \cdot \frac{x-3}{x-3} - \frac{-2x-1}{(x-1)(x-3)} \cdot \frac{2}{2}$$ LCD is $2(x-1)(x-3)$.

$$= \frac{x^2-x-6}{2(x-1)(x-3)} - \frac{-4x-2}{2(x-1)(x-3)}$$ Multiply.

$$= \frac{x^2-x-6-(-4x-2)}{2(x-1)(x-3)}$$ Subtract numerators.

$$= \frac{x^2+3x-4}{2(x-1)(x-3)}$$ Simplify numerator.

$$= \frac{\cancel{(x-1)}(x+4)}{2\cancel{(x-1)}(x-3)}$$ Factor numerator. Divide out common factors.

$$= \frac{x+4}{2(x-3)}, x \neq -1$$ Simplify.

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5. Find the least common multiple of $5x^3$ and $10x^2 - 15x$.

Find the sum or difference.

6. $\frac{3}{4x} - \frac{1}{7}$

7. $\frac{1}{3x^2} + \frac{x}{9x^2-12}$

8. $\frac{x}{x^2-x+12} + \frac{5}{12x-48}$

Rewriting Rational Functions

Rewriting a rational expression may reveal properties of the related function and its graph. In Example 4 of Section 7.2, you used long division to rewrite a rational expression. In the next example, you will use inspection.

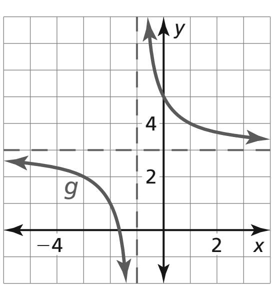
EXAMPLE 5 Rewriting and Graphing a Rational Function

Rewrite the function $g(x) = \frac{3x+5}{x+1}$ in the form $g(x) = \frac{a}{x-h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \frac{a}{x}$.

SOLUTION

Rewrite by inspection:

$$\frac{3x+5}{x+1} = \frac{3x+3+2}{x+1} = \frac{3(x+1)+2}{x+1} = \frac{3\cancel{(x+1)}}{\cancel{x+1}} + \frac{2}{x+1} = 3 + \frac{2}{x+1}$$



▶ The rewritten function is $g(x) = \frac{2}{x+1} + 3$. The graph of g is a translation 1 unit left and 3 units up of the graph of $f(x) = \frac{2}{x}$.

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9. Rewrite $g(x) = \frac{2x-4}{x-3}$ in the form $g(x) = \frac{a}{x-h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \frac{a}{x}$.

Complex Fractions

A **complex fraction** is a fraction that contains a fraction in its numerator or denominator. A complex fraction can be simplified using either of the methods below.

Core Concept

Simplifying Complex Fractions

Method 1 If necessary, simplify the numerator and denominator by writing each as a single fraction. Then divide by multiplying the numerator by the reciprocal of the denominator.

Method 2 Multiply the numerator and the denominator by the LCD of *every* fraction in the numerator and denominator. Then simplify.

EXAMPLE 6 Simplifying a Complex Fraction

Simplify $\dfrac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}}$.

SOLUTION

Method 1

$$\frac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}} = \frac{\frac{5}{x+4}}{\frac{3x+8}{x(x+4)}}$$ Add fractions in denominator.

$$= \frac{5}{x+4} \cdot \frac{x(x+4)}{3x+8}$$ Multiply by reciprocal.

$$= \frac{5x\cancel{(x+4)}}{\cancel{(x+4)}(3x+8)}$$ Divide out common factors.

$$= \frac{5x}{3x+8},\ x \neq -4,\ x \neq 0$$ Simplify.

Method 2 The LCD of all the fractions in the numerator and denominator is $x(x + 4)$.

$$\frac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}} = \frac{\frac{5}{x+4}}{\frac{1}{x+4}+\frac{2}{x}} \cdot \frac{x(x+4)}{x(x+4)}$$ Multiply numerator and denominator by the LCD.

$$= \frac{\frac{5}{\cancel{x+4}} \cdot x\cancel{(x+4)}}{\frac{1}{\cancel{x+4}} \cdot x\cancel{(x+4)} + \frac{2}{\cancel{x}} \cdot \cancel{x}(x+4)}$$ Divide out common factors.

$$= \frac{5x}{x+2(x+4)}$$ Simplify.

$$= \frac{5x}{3x+8},\ x \neq -4,\ x \neq 0$$ Simplify.

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Simplify the complex fraction.

10. $\dfrac{\frac{x}{6}-\frac{x}{3}}{\frac{x}{5}-\frac{7}{10}}$

11. $\dfrac{\frac{2}{x}-4}{\frac{2}{x}+3}$

12. $\dfrac{\frac{3}{x+5}}{\frac{2}{x-3}+\frac{1}{x+5}}$