

Limits - Chapter 2

$$1) \lim_{x \rightarrow 2} \frac{x^2 - 1}{x^3 + 2} = \frac{4 - 1}{8 + 2} = \boxed{\frac{3}{10}}$$

$$2) \lim_{x \rightarrow -2} \frac{x+2}{x^2-4} = \lim_{x \rightarrow -2} \frac{\cancel{(x+2)} \cdot 1}{\cancel{(x+2)}(x-2)}$$

$$= \lim_{x \rightarrow -2} \frac{1}{x-2} = \boxed{-\frac{1}{4}}$$

$$3) \lim_{x \rightarrow -2} \frac{x^2-4}{x^3+8} = \frac{4-4}{-8+8} = \frac{0}{0} \Rightarrow \lim_{x \rightarrow -2} \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x^2-2x+4)} = \lim_{x \rightarrow -2} \frac{x-2}{x^2-2x+4} = \frac{-4}{12} = \boxed{-\frac{1}{3}}$$

$$\begin{array}{r} -2 \overline{) 1008} \\ \underline{-24} \\ 1-24 \\ \underline{1-24} \\ 0 \end{array}$$

$$4) \lim_{x \rightarrow -1} \frac{\frac{1}{x+2} - 1}{x+1} = \lim_{x \rightarrow -1} \frac{\frac{1}{x+2} - \frac{x+1}{x+1}}{x+1} = \lim_{x \rightarrow -1} \frac{1 - (x+2)(x+1)}{(x+1)(x+2)}$$

$$= \lim_{x \rightarrow -1} \frac{-1 - x - 2 - x^2 - 3x - 2}{(x+1)(x+2)} = \lim_{x \rightarrow -1} \frac{-x^2 - 4x - 5}{(x+1)(x+2)} = \lim_{x \rightarrow -1} \frac{-1(x+4)(x+1)}{(x+1)(x+2)}$$

$$= \lim_{x \rightarrow -1} \frac{-1(x+4)}{(x+2)} = \frac{-3}{1} = \boxed{-3}$$

$$5) \lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{x} \cdot \frac{(\sqrt{x+9} + 3)}{(\sqrt{x+9} + 3)} = \lim_{x \rightarrow 0} \frac{(x+9) - 9}{x(\sqrt{x+9} + 3)} = \lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+9} + 3)}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\sqrt{x+9} + 3} = \boxed{\frac{1}{6}}$$

$$6) \lim_{x \rightarrow 0} \frac{\frac{1}{x+1} - 1}{x} = \lim_{x \rightarrow 0} \frac{\frac{1}{x+1} - \frac{x}{x}}{x} = \lim_{x \rightarrow 0} \frac{1 - (x+1) - x(x+1)}{x}$$

$$\lim_{x \rightarrow 0} \frac{1 - x - 1 - x^2 - x}{x} = \lim_{x \rightarrow 0} \frac{-x^2 - 2x}{x} = \lim_{x \rightarrow 0} \frac{-x(x+2)}{x} = \lim_{x \rightarrow 0} -(x+2) = \boxed{-2}$$

$$7) \lim_{x \rightarrow 1} \frac{\sqrt{3} - \sqrt{x+2}}{(1-x)} \cdot \frac{(\sqrt{3} + \sqrt{x+2})}{(\sqrt{3} + \sqrt{x+2})} = \lim_{x \rightarrow 1} \frac{3 - (x+2)}{(1-x)(\sqrt{3} + \sqrt{x+2})} = \lim_{x \rightarrow 1} \frac{1-x}{(1-x)(\sqrt{3} + \sqrt{x+2})}$$

$$= \lim_{x \rightarrow 1} \frac{1}{(\sqrt{3} + \sqrt{x+2})} = \frac{1}{2\sqrt{3}} = \boxed{\frac{\sqrt{3}}{6}}$$

$$8. \lim_{x \rightarrow -2} \frac{x^2 - 1}{2x} = \frac{4 - 1}{-4} = \boxed{-\frac{3}{4}}$$

$$9. \lim_{x \rightarrow c} [f(x)]^3 = 4^3 = \boxed{64}$$

$$10. \lim_{x \rightarrow c} [3f(x) - g(x)] = 3 \cdot 4 - 5 = \boxed{7}$$

$$11. \lim_{x \rightarrow c} [f(x) \cdot g(x)] = 4 \cdot 5 = \boxed{20}$$

$$12. \lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{4}{5}$$

$$13. \lim_{x \rightarrow c} \sqrt[3]{f(x)} = \sqrt[3]{27} = \boxed{3}$$

$$14. \lim_{x \rightarrow c} \frac{f(x)}{18} = \frac{27}{18} = \boxed{\frac{3}{2}}$$

$$15. \lim_{x \rightarrow c} [f(x) \cdot g(x)] = 27(12) = \boxed{324}$$

$$16. \lim_{x \rightarrow c} [f(x) - 2g(x)] = 27 - 2(12) = \boxed{3}$$

$$17. \lim_{x \rightarrow -1^+} (-|x+1| - 1) = \boxed{-1}$$

$$18. \lim_{x \rightarrow -2^-} (x + |2x+4|) = \boxed{-2}$$

$$19. \lim_{x \rightarrow 2^+} f(x) = -2(2) + 1 = \boxed{-3}$$

$$20. \lim_{x \rightarrow \frac{\pi}{2}^-} 2 \sec x = \boxed{\infty}$$

$$21. \lim_{x \rightarrow 2^+} \frac{x-2}{x^2-3x+2} = \lim_{x \rightarrow 2^+} \frac{x-2}{(x-2)(x-1)} = \boxed{1}$$

$$22. \lim_{x \rightarrow 1^+} \frac{x-1}{x^2-4x+3} = \lim_{x \rightarrow 1^+} \frac{x-1}{(x-1)(x-3)} = \boxed{\frac{-1}{2}}$$



$$23. \lim_{x \rightarrow 2^+} f(x) = \frac{2}{2} + 2 = \boxed{3}$$

$$24. \lim_{x \rightarrow \frac{\pi}{3}^+} -\cot(2x) = -\cot\left(\frac{2\pi}{3}\right) = \boxed{\frac{\sqrt{3}}{3}}$$

$$25. f(x) = \sin\left(\frac{1}{x-\pi}\right)$$

Cont on $(-\infty, \pi) \cup (\pi, \infty)$

$$26. f(x) = \begin{cases} x+3, & x \leq -4 \\ -x-9, & x > -4 \end{cases}$$

Cont on $(-\infty, -4) \cup (-4, \infty)$

$$27. f(x) = -2\cot(2x) \text{ over } [-\pi, \pi] \\ = \frac{-2\cos(2x)}{\sin(2x)}$$

$$\sin 2x = 0 \Rightarrow 2x = -\pi, 0, \pi, -2\pi, 2\pi \\ \Rightarrow x = -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi$$

Cont on $(-\infty, \infty) \cap (x \neq -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi)$

$$28. f(x) = \begin{cases} -2x+6, & x < 6 \\ -2, & x \geq 6 \end{cases}$$

Cont on $(-\infty, 6) \cup (6, \infty)$

$$29. f(x) = \frac{x+1}{x^2-4x} = \frac{x+1}{x(x-4)}$$

Cont on $(-\infty, 0) \cup (0, 4) \cup (4, \infty)$

$$31. f(x) = \begin{cases} -\frac{x}{2} - 1, & x \leq 1 \\ -x^2 + 4x - 3, & x > 1 \end{cases}$$

Cont on $(-\infty, 1) \cup (1, \infty)$

$$33. f(x) = \begin{cases} x^2 - 4x + 3, & x \geq 3 \\ -2x + 3, & x < 3 \end{cases}$$

$$\lim_{x \rightarrow 3^+} f(x) = 9 - 12 + 3 = 0$$

$$\lim_{x \rightarrow 3^-} f(x) = -6 + 3 = 3$$

$$0 \neq 3$$

Discont @ $x = 3$
Jump discont.

$$35. f(x) = \frac{-x-1}{x^2+x+1}$$

$$x^2+x+1 \neq 0$$

Cont $(-\infty, \infty)$

$$37. f(x) = \sin\left(\frac{1}{x+\pi}\right)$$

Discont @ $x = -\pi$
oscillates

$$30. f(x) = \frac{25}{x^2+25} \quad x^2+25 \neq 0$$

Cont $(-\infty, \infty)$

$$32. f(x) = \begin{cases} \frac{x}{2} - 3, & x > -4 \\ -x^2 + 4x - 5, & x \leq -4 \end{cases}$$

Cont on $(-\infty, -4) \cup (-4, \infty)$

$$34. f(x) = \frac{-x^2 - 9x + 18}{x-1} = -\frac{(x-3)(x-6)}{x-1}$$

Discont @ $x = 1$
 $\lim_{x \rightarrow 1} f(x) = \text{DNE}$
Vert. Asymptote

$$36. f(x) = \cos\left(\frac{1}{x}\right) \text{ over } [-\pi, \pi]$$

Discont @ $x = 0$
 $f(x)$ oscillates about $x = 0$

$$38. f(x) = -\tan(2x) \text{ over } [-\pi, \pi]$$

$$= -\frac{\sin 2x}{\cos 2x}$$

$$\cos 2x = 0 \Rightarrow 2x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\Rightarrow x = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$$

Discont @ $x = \pm\frac{3\pi}{4}, \pm\frac{\pi}{4}$
Vert. Asy

$$39. f(x) = \frac{x^2 - 2x - 8}{x+2} = \frac{(x-4)\cancel{(x+2)}}{\cancel{x+2}} = x-4; x \neq -2$$

Discont @ $x = -2$ Removable Pt of Discont

$$40. f(x) = \begin{cases} -x^2 + 16x - 63, & x \geq 7 & -49 + 112 - 63 = 0 \\ -2x + 9, & x < 7 & -14 + 9 = -5 \end{cases}$$

Discont @ $x = 7$ Jump Discont.