Calculus
Chapter 4 – Final Review

Learning Target: I can use the First Derivative Test to find characteristics of a function including extrema.

For each function, a) find the critical points (if any), b) find the open intervals on which the function is increasing or decreasing, and c) apply the First Derivative Test to identify relative extrema.

1) $f(x) = x^2 - 6x$

2) $f(x) = x^2(3-x)$

3) $f(x) = \frac{x+3}{x^2}$

4) $f(x) = x + \frac{1}{x}$

Consider each of the following functions on the interval $(0, 2\pi)$, then a) find the critical points (if any), b) find the open intervals on which the function is increasing or decreasing, and c) apply the First Derivative Test to identify relative extrema.

5) $f(x) = \frac{x}{2} + \cos x$

6) $f(x) = \sin x + \cos x$

7) $f(x) = \cos^2(2x)$

8) $f(x) = \frac{\sin x}{1+\cos^2 x}$

Learning Target: I can use Second Derivative Test to determine characteristics of a function including intervals of concavity and points of inflection.

Determine the open intervals on which the graph is concave upward or concave downward.

9) $f(x) = -x^3 + 3x^2 - 2$

10) $f(x) = \frac{x^2 - 1}{2x+1}$

11) $f(x) = x + \frac{2}{\sin x}, \quad (-\pi, \pi)$

Find the points of inflection and discuss the concavity of the graph of the function.

12) $f(x) = x\sqrt{x+3}$

13) $f(x) = 2\csc\frac{3x}{2}, \quad [0, 2\pi]$

14) $f(x) = \frac{x+1}{\sqrt{x}}$

Find all relative extrema. Use the Second Derivative Test when applicable.

15) $f(x) = (5-x)^2$

16) $f(x) = x^{2/3} - 3$

17) $f(x) = \sqrt{x^2+1}$

Learning Target: I can use derivatives to solve optimization problems.

18) A farmer plans to fence a rectangular pasture adjacent to a river. The [pasture must contain 180,000 square meters in order to provide enough grass for the herd. What dimensions would require the least amount of fencing if no fencing is needed along the river?

19) Determine the dimensions of a rectangular solid (with a square base) with maximum volume if its surface area is 337.5 square centimeters.

20) A rectangle is bounded by the x-axis and the semicircle $y=\sqrt{25-x^2}$. What length and width should the rectangle have so that its area is a maximum?

21) An offshore oil well is 2 kilometers off the coast. The refinery is 4 kilometers down the coast. Laying pipe in the ocean is twice as expensive as on land. What path should the pipe follow in order to minimize the cost?

22) An open box is to be made from a square piece of material, 24 inches on a side, by cutting equal squares from the corners and turning up the sides. What dimensions should the box have to maximize its volume? What is the maximum volume?

Learning target: I can use derivatives to solve related rates problems.

23) The radius r of a circle is increasing at a rate of 3 centimeters per minute. Find the rates of change of the area when r = 6 centimeters and when r = 24 centimeters.

24) A spherical balloon is inflated with gas at the rate of 800 cubic centimeters per minute. How fast is the radius increasing at the instant the radius is 30 centimeters and when the radius is 60 centimeters?

25) At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at a rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 15 feet high?

26) A baseball diamond has the shape of a square with sides 90 feet long. A player running from second to third base at a speed of 28 feet per second is 30 feet from third base. At what rate is the player's distance from home plate changing?

27) An airplane is flying in still air with an airspeed of 240 miles per hour. If it is climbing at an angle of 22°, find the rate at which it is gaining elevation.