Calculus
Mixed Practice #2

Name:
Date:
Period:

1. $\int_{-\infty}^{0} \frac{e^x}{3-2e^x}dx$

2. $\int x^3 \cos\left(2x^4-1\right)dx$

3. $\lim_{x\to 1} \frac{\ln x}{\tan \pi x}$

4. Which grows faster as $x\to\infty$ $\tan x$ and x^2?

5. $\frac{d}{dx}\left[\left(x^3-\frac{7}{x}\right)^{-2}\right]$

6. Which grows faster as $x\to\infty$ $\ln x$ and $\sqrt[3]{x}$?

7. $\int_0^{\frac{\pi}{2}} \cos^4 2x \sin 2x dx$

8. $\frac{d}{dx}\left[x^2\sqrt{5-x^2}\right]$

9. $\lim_{x\to\infty} xe^{-x}$

10. $\frac{d}{dx}\left[\cos^3\left(\frac{x}{x+1}\right)\right]$

11. $\lim_{x\to 0}\left(1+2x\right)^{\frac{-3}{x}}$

12. $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)$

13. $\int_0^{\infty} \frac{x}{\left(x^2+1\right)^2}dx$

14. $\frac{d}{dx}\left[x\sin 3x\right]$

15. $\lim_{x\to\pi^+} \frac{\sin x}{x-\pi}$

16. $\int_{-1}^{4} \frac{1+e^x}{e^x}dx$

17. $\int_{-1}^{8} \frac{dx}{\sqrt[3]{x}}$

18. $\int x^e dx$

19. $\int_0^3 \frac{dx}{x-2}$

20. $\int_{-\infty}^{\infty} \frac{dx}{\left(x-2\right)^{\frac{2}{3}}}$

Mixed Review #2

1. $\int_{-\infty}^{0} \frac{e^x}{3-2e^x}\,dx = \lim_{a\to-\infty} -\frac{1}{2}\int_a^0 \frac{-2e^x}{3-2e^x}\,dx = \lim_{a\to-\infty} -\frac{1}{2}\int_a^0 \frac{-2e^x}{3-2e^x}\,dx$

$u = 3-2e^x$
$du = -2e^x\,dx$

$= \lim_{a\to-\infty} -\frac{1}{2}\left(\ln|3-2e^x|\right)\Big|_a^0 = -\frac{1}{2}\left(\cancel{\ln|1|} - \ln|3|\right)$

$\boxed{= \frac{1}{2}\ln 3}$

2. $\frac{1}{8}\int 8x^3\cos(2x^4-1)\,dx \Rightarrow \frac{1}{8}\int \cos u\cdot du = \frac{1}{8}\sin u + C$

$u = 2x^4-1$
$du = 8x^3\,dx$

$\Rightarrow \boxed{\frac{1}{8}\sin(2x^4-1)+C}$

3. $\lim_{x\to1} \frac{\ln x}{\tan\pi x}\ \frac{0}{0} = \lim_{x\to1} \frac{\frac{1}{x}}{\pi\sec^2\pi x} = \lim_{x\to1} \frac{\cos^2\pi x}{\pi x} = \frac{\cos^2\pi}{\pi}$

note: $\frac{1}{\sec} = \cos$

$= \frac{(-1)^2}{\pi} = \boxed{\frac{1}{\pi}}$

4. $\lim_{x\to\infty} \frac{\tan x}{x^2} = \lim_{x\to\infty} \frac{\sec^2 x}{2x} = \lim_{x\to\infty} \frac{2\sec x\cdot\sec x\cdot\tan x}{2}$

note: $\tan x = \frac{\sin x}{\cos x}$

$= \lim_{x\to\infty} \frac{\cancel{2}\sec^2 x\cdot\sin x}{\cancel{2}\cos x} = \lim_{x\to\infty} \frac{\sin x}{\cos^3 x}$

note: $\sec^2 x = \frac{1}{\cos^2 x}$

$\boxed{\text{limit DNE} \therefore \text{Cannot be determined.}}$

5. $\frac{d}{dx}\left(x^3-\frac{7}{x}\right)^{-2} = \boxed{-2\left(x^3-\frac{7}{x}\right)^{-3}\left(3x^2+\frac{7}{x^2}\right)}$

6. $\lim_{x\to\infty} \frac{\ln x}{\sqrt[3]{x}} = \lim_{x\to\infty} \frac{\ln x}{x^{1/3}} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{3}x^{-2/3}} = \lim_{x\to\infty} \frac{3x^{2/3}}{x}$

$= \lim_{x\to\infty} \frac{3}{x^{1/3}} = 0$ $\boxed{\text{So } \sqrt[3]{x} \text{ grows faster than } \ln x.}$

7. $-\frac{1}{2}\int_0^{\pi/2} -2\cos^4 2x \sin 2x \, dx \Rightarrow -\frac{1}{2}\int u^4 \, du = -\frac{1}{2}\cdot\frac{u^5}{5} + C$

$u = \cos 2x$

$du = -2\sin 2x \, dx$

$= \frac{-u^5}{10} + C$

$\Rightarrow -\frac{\cos^5 2x}{10}\Big|_0^{\pi/2} = -\frac{\cos^5 \pi}{10} + \frac{\cos^5(0)}{10}$

$= \frac{-(-1)^5}{10} + \frac{1^5}{10} = \frac{1}{10} + \frac{1}{10} = \boxed{\frac{1}{5}}$

8. $\frac{d}{dx}\left[\overset{u}{x^2}\overset{v}{\sqrt{5-x^2}}\right] = x^2 \cdot \frac{-x}{\sqrt{5-x^2}} + \sqrt{5-x^2} \cdot 2x$

$du = 2x\,dx \quad dv = \frac{1}{2}(5-x^2)^{-1/2}(-2x)dx$

$= -x(5-x^2)^{-1/2}$

$= \frac{-x}{\sqrt{5-x^2}}$

find a common denom.

$= \frac{-x^3}{\sqrt{5-x^2}} + \sqrt{5-x^2} \cdot 2x = \frac{-x^3 + (5-x^2)\cdot 2x}{\sqrt{5-x^2}} = \frac{-x^3 + 10x^2 - 2x}{\sqrt{5-x^2}}$

$= \boxed{\frac{-3x^3 + 10x^2}{\sqrt{5-x^2}}}$

9. $\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty} \frac{x}{e^x} = \lim_{x\to\infty} \frac{1}{e^x} = \boxed{0}$

10. $\frac{d}{dx}\left[\cos^3\left(\frac{x}{x+1}\right)\right] = 3\cos^2\left(\frac{x}{x+1}\right)\left(-\sin\frac{x}{x+1}\right)\left(\frac{(x+1)-x}{(x+1)^2}\right)$

10. $= \boxed{\dfrac{-3}{(x+1)^2}\cos^2\left(\dfrac{x}{x+1}\right)\sin\left(\dfrac{x}{x+1}\right)}$

11. $\lim\limits_{x\to 0} (1+2x)^{-3/x}$

$y = (1+2x)^{-3/x}$

$\ln y = -\dfrac{3}{x} \ln(1+2x)$

$\ln \lim\limits_{x\to 0} y = \lim\limits_{x\to 0} \dfrac{-3}{x} \ln(1+2x)$

$= \lim\limits_{x\to 0} \dfrac{-3\ln(1+2x)}{x}$

$= \lim\limits_{x\to 0} \dfrac{\frac{-3}{1+2x}\cdot 2}{1}$

$= \lim\limits_{x\to 0} \dfrac{-6}{1+\cancel{2x}_{\,0}} = -6$

So, $e^{\ln \lim\limits_{x\to 0} y} = e^{-6}$

$\lim\limits_{x\to 0} y = \boxed{e^{-6} = \dfrac{1}{e^6}}$

12. $\lim\limits_{x\to 0} \left(\dfrac{1}{x} - \dfrac{1}{e^x-1}\right) \overset{\text{find a common denom.}}{=} \lim\limits_{x\to 0} \dfrac{e^x-1-x}{x(e^x-1)}$

$= \lim\limits_{x\to 0} \dfrac{e^x-1}{x(e^x)+(e^x-1)} = \lim\limits_{x\to 0} \dfrac{e^x}{xe^x+e^x+e^x} = \lim\limits_{x\to 0} \dfrac{\cancel{e^x}\ 1}{\cancel{e^x}(x+2)}$

$= \lim\limits_{x\to 0} \dfrac{1}{\cancel{x}_{\,0}+2} = \boxed{\dfrac{1}{2}}$

$$\int \frac{du}{u^2} = \int u^{-2}\,du = -u^{-1}+c = \frac{-1}{u}+c$$

13. $\displaystyle\int_0^\infty \frac{x}{(x^2+1)^2}\,dx = \lim_{a\to\infty} \frac{1}{2}\int_0^a \frac{2x}{(x^2+1)^2}\,dx = \lim_{a\to\infty} \frac{1}{2} \cdot \frac{-1}{x^2+1}\Big|_0^a$

$u = x^2+1$
$du = 2x\,dx$

$\displaystyle = \lim_{a\to\infty} \cancel{\frac{-1}{2(a^2+1)}} + \frac{1}{2(0^2+1)} = \boxed{\frac{1}{2}}$

0

∞

14. $\displaystyle \frac{d}{dx}\left(\overset{u}{x}\overset{v}{\sin 3x}\right) = x(3\cos 3x) + \sin 3x\,(1) = \boxed{3x\cos 3x + \sin 3x}$

$du = 1 \quad dv = 3\cos 3x$

15. $\displaystyle \lim_{x\to\pi^+} \frac{\sin x}{x-\pi}\ \frac{0}{0} = \lim_{x\to\pi^+} \frac{\cancel{\cos x}}{1} = \boxed{-1}$

-1

16. $\displaystyle \int_{-1}^4 \frac{1+e^x}{e^x}\,dx = \int_1^4 \left(\frac{1}{e^x}+1\right)dx = \int_1^4 (e^{-x}+1)\,dx$

$\displaystyle = -1\int_1^4 -e^{-x}\,dx + \int_1^4 1\,dx = (-e^{-x}+x)\Big|_1^4 = -e^{-4}+4-(-e^{-1}+1)$

$\displaystyle = \frac{-1}{e^4}+4+\frac{1}{e}-1 = \boxed{3+\frac{1}{e}-\frac{1}{e^4} \text{ or } 3+e^{-1}-e^{-4} \text{ or } \frac{3e^4+e^3-1}{e^4}}$

17. $\displaystyle \int_{-1}^8 \frac{dx}{\sqrt[3]{x}} = \int_{-1}^0 x^{-1/3}\,dx + \int_0^8 x^{-1/3}\,dx = \lim_{a\to 0}\int_{-1}^a x^{-1/3}\,dx + \lim_{b\to 0}\int_b^8 x^{-1/3}\,dx$

$\displaystyle = \lim_{a\to 0} \frac{3}{2}x^{2/3}\Big|_{-1}^a + \lim_{b\to 0}\frac{3}{2}x^{2/3}\Big|_b^8 = \cancel{\lim_{a\to 0}\frac{3}{2}a^{2/3}} - \frac{3}{2}(-1)^{2/3} + \frac{3}{2}(8)^{2/3}$

0

$\displaystyle -\cancel{\lim_{b\to 0}\frac{3}{2}b^{2/3}} = -\frac{3}{2}+\frac{3}{2}(4) = -\frac{3}{2}+6 = \boxed{\frac{9}{2}}$

0

18. $\int x^e\,dx = \boxed{\dfrac{x^{e+1}}{e+1} + C}$

19. $\displaystyle\int_0^3 \frac{dx}{x-2} = \int_0^2 \frac{dx}{x-2} + \int_2^3 \frac{dx}{x-2}$

$\displaystyle = \lim_{a\to 2^-} \int_0^a \frac{dx}{x-2} + \lim_{b\to 2^+} \int_b^3 \frac{dx}{x-2}$

$\displaystyle = \lim_{a\to 2^-} \ln|x-2|\Big|_0^a + \lim_{b\to 2^+} \ln|x-2|\Big|_b^3$

$\displaystyle = \overset{-\infty}{\cancel{\lim_{a\to 2^-} \ln|a-2|}} - \ln|-2| + \ln|3-2| - \overset{-\infty}{\cancel{\lim_{b\to 2^+} \ln|b-2|}}$

$\boxed{= \infty \quad \text{Diverges}}$

20. $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x-2)^{2/3}} = \int_{-\infty}^{2} \frac{dx}{(x-2)^{2/3}} + \int_2^{\infty} \frac{dx}{(x-2)^{2/3}}$

$\displaystyle = \int_{-\infty}^{0} \frac{dx}{(x-2)^{2/3}} + \int_0^2 \frac{dx}{(x-2)^{2/3}} + \int_2^3 \frac{dx}{(x-2)^{2/3}} + \int_3^{\infty} \frac{dx}{(x-2)^{2/3}}$

$\displaystyle \lim_{a\to -\infty} \int_a^0 (x-2)^{-2/3}\,dx + \lim_{b\to 2^-} \int_0^b (x-2)^{-2/3}\,dx + \lim_{c\to 2^+} \int_b^3 (x-2)^{-2/3}\,dx + \lim_{d\to\infty} \int_3^d (x-2)^{-2/3}$

$\displaystyle = \lim_{a\to -\infty} 3(x-2)^{1/3}\Big|_a^0 + \lim_{b\to 2^-} 3(x-2)^{1/3}\Big|_0^b + \lim_{c\to 2^+} 3(x-2)^{1/3}\Big|_b^3 + \lim_{d\to\infty} 3(x-2)^{1/3}\Big|_3^d$

$\displaystyle = 3(-2)^{1/3} - \overset{-\infty}{\cancel{\lim_{a\to -\infty} 3(a-2)^{1/3}}} + \ldots$

Because 1 limit goes to $\pm\infty$, the entire integral diverges. There is no need to finish the problem.