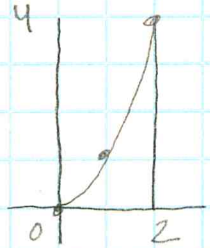
ch 5

1. $f(x)=x^2$



$a=0$
$\Delta x=\frac{2}{n}$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}f\left(0+\frac{2}{n}i\right)$$

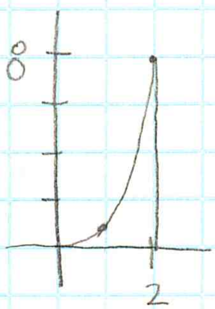
$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\left(\frac{2}{n}i\right)^2=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{8}{n^3}i^2$$

$$=\lim_{n\to\infty}\frac{8}{n^3}\sum_{i=1}^{n}i^2=\lim_{n\to\infty}\frac{8}{n^3}\cdot\frac{n\overbrace{(n+1)(2n+1)}^{2n^2+3n+1}}{6}$$

$$=\lim_{n\to\infty}\frac{8}{n^3}\left(\frac{2n^3+3n^2+n}{6}\right)$$

$$=\lim_{n\to\infty}\frac{3}{4}\left(\frac{2n^3}{n^3}+\overset{0}{\cancel{\frac{3n^2}{n^3}}}+\overset{0}{\cancel{\frac{n}{n^3}}}\right)=\frac{6}{4}=\frac{3}{2}$$

2. $f(x)=8-x^3$



$\Delta x=\frac{2}{n}$
$a=0$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\left(8-\left(\frac{2}{n}i\right)^3\right)=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\left(8-\frac{8}{n^3}i^3\right)$$

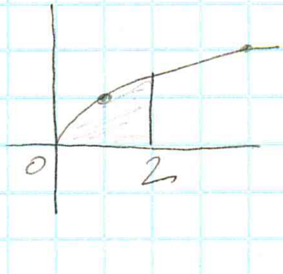
$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{16}{n}-\frac{16}{n^4}i^3\right)=\lim_{n\to\infty}\sum_{i=1}^{n}\frac{16}{n}-\lim_{n\to\infty}\sum_{i=1}^{n}\frac{16}{n^4}i^3$$

$$=\lim_{n\to\infty}16-\lim_{n\to\infty}\frac{16}{n^4}\cdot\frac{n^2(n+1)^2}{4}$$

$$=16-\lim_{n\to\infty}\frac{4}{n^2}(n^2+2n+1)=16-\lim_{n\to\infty}\left(4+\overset{0}{\cancel{\frac{8}{n}}}+\overset{0}{\cancel{\frac{4}{n^2}}}\right)$$

$$=16-4=12$$

3. $f(x)=\sqrt{x}$ $n=4$ R

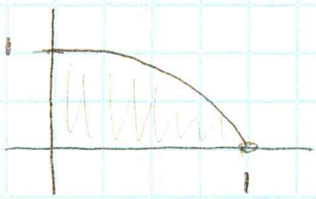


$\Delta x=\frac{2}{4}=\frac{1}{2}$

$$\sum_{i=1}^{4}\frac{1}{2}\sqrt{0+\frac{1}{2}i}=\frac{1}{2}(.707+1+1.225+1.4$$

$$\boxed{=2.173}$$

4. $f(x)=\sqrt{1-x^2}$ $\quad n=8\ L$



$\Delta x=\frac{1}{8}$

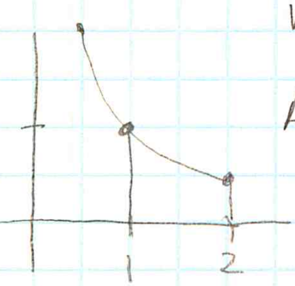
$a=0$

$$\sum_{i=0}^{7}\frac{1}{8}\sqrt{1-\left(\frac{i}{8}\right)^2}=\sum_{i=0}^{7}\frac{1}{8}\sqrt{1-\frac{i^2}{64}}$$

$$=\frac{1}{8}\left(1+.99216+.96825+\dots+.66144\right)$$

$$\boxed{=.83495}$$

5. $f(x)=\frac{1}{x}$



$n=5R$

$\Delta x=\frac{1}{5}$

$a=1$

$$\sum_{i=1}^{5}\frac{1}{5}\cdot\frac{1}{1+\frac{i}{5}}=\sum_{i=1}^{5}\frac{1}{\cancel{5}}\cdot\frac{\cancel{5}}{5+i}=\sum_{i=1}^{5}\frac{1}{5+i}$$

$$\boxed{=.6456}$$

1) $$\int_0^4\sqrt{x}\,dx=\frac{2}{3}x^{3/2}\Big|_0^4=\frac{2}{3}\left(2^3-0\right)=\frac{16}{3}$$

2) $$\frac{1}{\pi}\int_0^{1/2}\pi 4\cos\pi x\,dx=\frac{4}{\pi}\sin\pi x\Big|_0^{1/2}=\frac{4}{\pi}\left(\cancel{\sin\pi/2}^{\,1}-\cancel{\sin 0}^{\,0}\right)=\frac{4}{\pi}$$

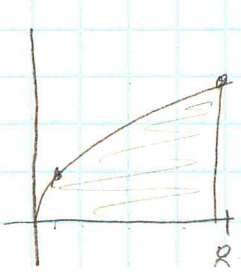
3) $$\int_0^3\frac{5}{x^2+1}\,dx=5\tan^{-1}x\Big|_0^3=5\tan^{-1}(3)-5\tan^{-1}(0)\boxed{=6.245}$$

4) $$\int_0^1(x-x^2)\,dx=\frac{x^2}{2}-\frac{x^3}{3}\Big|_0^1=\frac{1}{2}-\frac{1}{3}-0=\boxed{\frac{1}{6}}$$

5) $$\int_0^3(3-x)\sqrt{x}\,dx=\int_0^3\left(3\sqrt{x}-x^{3/2}\right)dx=\left(\cancel{3}\frac{2}{\cancel{3}}x^{3/2}-\frac{2}{5}x^{5/2}\right)\Big|_0^3$$

$$=2(3)^{3/2}-\frac{2}{5}(3)^{5/2}-0\boxed{=4.157}$$

6) $f(x)=1+\sqrt[3]{x}$



$$\int_0^8\left(1+\sqrt[3]{x}\right)dx=\left(x+\frac{3}{4}x^{4/3}\right)\Big|_0^8=8+\frac{3}{4}2^4-0$$

$$=8+\frac{3}{4}\cdot16=8+12\boxed{=20}$$