Ch 6

1) $\frac{1}{2}\int_{-1}^{2} 2x(x^2-4)\,dx = \frac{1}{2}\frac{(x^2-4)^2}{2}\Big|_{-1}^{2} = \frac{(x^2-4)^2}{4}\Big|_{-1}^{2} = \frac{0}{4} - \left(\frac{(1-4)^2}{4}\right) =$

2) $\int (x^2+1)^3\,dx = \int\left((x^2)^3 + 3(x^2)^2(1) + 3(x^2)(1)^2 + 1^3\right)dx$

$= \int (x^6 + 3x^4 + 3x^2 + 1)\,dx = \boxed{\frac{x^7}{7} + \frac{3x^5}{5} + x^3 + x + C}$

3) $\frac{1}{3}\int_0^1 3x^2(x^3+1)^3\,dx = \frac{1}{3}\frac{(x^3+1)^4}{4}\Big|_0^1 = \frac{(x^3+1)}{12}\Big|_0^1 = \frac{2}{12} - \frac{1}{12} = \boxed{\frac{1}{12}}$

4) $\int \left(x + \frac{1}{x}\right)^2 dx = \int \left(x^2 + 2 + \frac{1}{x^2}\right)dx = \boxed{\frac{x^3}{3} + 2x - \frac{1}{x} + C}$

5) $\frac{1}{3}\int \frac{3x^2}{\sqrt{x^3+3}}\,dx = \frac{1}{3}\sqrt{x^3+3}\cdot 2 + C = \boxed{\frac{2}{3}\sqrt{x^3+3} + C}$

6) $\frac{-1}{6}\int -6x(1-3x^2)^4\,dx = \frac{-1}{6}\frac{(1-3x^2)^5}{5} + C = \boxed{-\frac{(1-3x^2)^5}{30} + C}$

7) $\frac{1}{2}\int \frac{2(x+3)}{(x^2+6x-5)^2}\,dx = \frac{-1}{2}\cdot\frac{1}{(x^2+6x-5)} + C = \boxed{\frac{-1}{2(x^2+6x-5)} + C}$

8) $\int \sin^3 x \cos x\,dx = \boxed{\frac{\sin^4 x}{4} + C}$

9) $\int xe^x\,dx = xe^x - \int e^x\,dx = \boxed{xe^x - e^x + C}$

$u = x \quad dv = e^x\,dx$

$du = 1dx \quad V = e^x$

10) $\int_0^3 \frac{1}{\sqrt{1+x}}\,dx = 2(1+x)^{\frac{1}{2}}\Big|_0^3 = 2(4)^{1/2} - 0 = \boxed{4}$

11) $\int x^4 \ln x\,dx = \frac{x^5 \ln x}{5} - \int \frac{x^{\cancel{5}\,4}}{5}\cdot\frac{1}{\cancel{x}}\,dx = \boxed{\frac{x^5 \ln x}{5} - \frac{x^5}{25} + C}$

$u = \ln x \quad dv = x^4\,dx$

$du = \frac{1}{x}\,dx \quad V = \frac{x^5}{5}$

12) $\frac{1}{2}\int_3^6 \frac{2x}{3\sqrt{x^2-8}}\,dx = \frac{1}{\cancel{2}}\cdot\frac{1}{3}\sqrt{x^2-8}\,\cancel{2}\Big|_3^6 = \frac{\sqrt{28}}{3} - \frac{1}{3} = 1.431$

13) $2\pi\int_0^1 (1+y)\sqrt{1-y}\,dy = 2\pi\left[(1+y)\left(\frac{-2}{3}(1-y)^{3/2}\right)\Big|_0^1 - \int_0^1 \frac{-2}{3}(1-y)^{3/2}\,dy\right]$

$u = 1+y \quad dv = \sqrt{1-y}\,dy$

$du = 1dy \quad V = \frac{-2}{3}(1-y)^{3/2}$ $= 2\pi\left[(1+y)\left(\frac{-2}{3}(1-y)^{3/2}\right) - \frac{2}{3}\cdot\frac{2}{5}(1-y)^{5/2}\right]\Big|_0^1$

$= 2\pi\left[2\left(\cancel{\frac{-2}{3}(0)}\right) - \cancel{\frac{4}{15}(0)} - \left[1\cdot\frac{-2}{3}\cdot 1 - \frac{4}{15}\cdot 1\right]\right]$

$= -2\pi\left(\frac{-2}{3} - \frac{4}{15}\right) = \left(\frac{2}{3} + \frac{4}{15}\right)2\pi = \boxed{\frac{28\pi}{15}}$

14) $\int x\cos x\,dx$

$u = x \quad dv = \cos x\,dx$

$du = dx \quad V = \sin x$

$= x\sin x - \int \sin x\,dx$

$= \boxed{x\sin x + \cos x + C}$

15) $\int \frac{\ln 2x}{x^2}\,dx$

$= -\frac{\ln 2x}{x} + \int \frac{1}{x^2}\,dx$

$u = \ln 2x \quad dv = x^{-2}\,dx$

$du = \frac{1}{x}\,dx \quad V = -x^{-1}$

$= \boxed{-\frac{\ln 2x}{x} - \frac{1}{x} + C}$

16) $\frac{1}{2}\int 2\cdot 3^{2x+1}\,dx = \boxed{\frac{3^{2x+1}}{\ln 3} + C}$

17) $2\pi\int_{-1}^0 x^2\sqrt{x+1}\,dx = 2\pi\left[\frac{2x^2(x+1)^{3/2}}{3} - \int \frac{4x}{3}(x+1)^{3/2}\,dx\right]$

$u = x^2 \quad dv = \sqrt{x+1}\,dx$

$du = 2x\,dx \quad V = \frac{2}{3}(x+1)^{3/2}$

$= 2\pi\left[\frac{2x^2(x+1)^{3/2}}{3} - \frac{4}{3}\left(\frac{2x(x+1)^{5/2}}{5} - \frac{2}{5}\int_{-1}^0 (x+1)^{5/2}\,dx\right)\right]$

$u = x \quad dv = (x+1)^{3/2}\,dx$

$du = dx \quad V = \frac{2}{5}(x+1)^{5/2}$

$= 2\pi\left[\frac{2x^2(x+1)^{3/2}}{3} - \frac{8x}{15}(x+1)^{5/2} + \frac{8}{15}\cdot\frac{2}{7}(x+1)^{7/2}\right]\Big|$

$= 2\pi\left[0\left(\cancel{\frac{1}{3}}\right) - \cancel{0(1)} + \frac{16}{105}\cdot 1 - \left(\cancel{\frac{2(0)}{3}} + \cancel{\frac{8}{15}(0)} + \cancel{\frac{16}{105}(0)}\right)\right] = \boxed{\frac{32\pi}{105}}$

18) $\int \frac{e^{4x}-e^{2x}+1}{e^{x}}\,dx = \int \left(e^{3x}-e^{x}+e^{-x}\right)dx = \frac{1}{3}\int 3e^{3x}\,dx - \int e^{x}\,dx \mp \int e^{-x}\,dx$

$= \boxed{\frac{e^{3x}}{3} - e^{x} - e^{-x} + C}$

19) $2\int_0^{\pi} \frac{1}{2}\cos\frac{x}{2}\,dx = 2\sin\left(\frac{x}{2}\right)\Big|_0^{\pi} = 2\sin\frac{\pi}{2} - \cancel{2\sin 0}^{\,0} = \boxed{2}$

20) $\frac{1}{2}\int_{-\pi/4}^{\pi/4} 2\sin 2x\,dx = -\frac{1}{2}\cos 2x\Big|_{-\pi/4}^{\pi/4} = \left(-\frac{1}{2}\cancel{\cos}^{\,0}\frac{\pi}{2} + \frac{1}{2}\cancel{\cos}^{\,0}\frac{\pi}{2}\right) = \boxed{0}$

21) $\int_0^{4} \frac{1}{\sqrt{25-x^2}}\,dx = \int_0^{4} \frac{1/5}{\sqrt{1-\left(\frac{x}{5}\right)^2}}\,dx = \sin^{-1}\left(\frac{x}{5}\right)\Big|_0^{4} = \sin^{-1}\left(\frac{4}{5}\right) - \sin^{-1}(0)$

$= \boxed{.9273}$

22) $\int_0^{2/3} \frac{1}{4+9x^2}\,dx = \frac{1}{4}\cdot\frac{2}{3}\int_0^{2/3} \frac{1\cdot 3/2}{1+\left(\frac{3x}{2}\right)^2}\,dx = \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)\Big|_0^{2/3}$

$= \frac{1}{6}\tan^{-1}(1) - \frac{1}{6}\tan^{-1}(0) = \boxed{.1309}$

23) $\int \frac{2}{(2t-1)^2+4}\,dt \Rightarrow \int \frac{du}{u^2+4} = \frac{1}{4}\int \frac{du}{\left(\frac{u}{2}\right)^2+1} = \frac{1}{4}\tan^{-1}\left(\frac{u}{2}\right) + C$

$u = 2t-1$
$du = 2$

$\Rightarrow \boxed{\frac{1}{4}\tan^{-1}\left(\frac{2t-1}{2}\right) + C}$

24) $\frac{1}{3}\int 3\left(\frac{1}{3x-1} - \frac{1}{3x+1}\right)dx = \frac{1}{3}\int\left(\frac{3}{3x-1} - \frac{3}{3x+1}\right)dx$

$= \boxed{\frac{1}{3}\ln|3x-1| - \frac{1}{3}\ln|3x+1| + C}$