

## How to Solve by Completing the Square:

when  $a = 1$

### Steps

- 1) Put equation in standard form ( $ax^2 + bx + c$ )
- 2) Divide coefficient of the x-term by 2 and square it.  
This is the number that actually completes the square.
- 3) Add AND subtract the number you found in step 2.  
Always add first.
- 4) Rewrite the perfect square (first three terms) as a squared term.
- 5) Simplify.
- 6) Solve for x.

### Example

$$x^2 - 12x = 3$$

$$x^2 - 12x - 3 = 0$$

$$\left(\frac{-12}{2} = -6\right)^2 = 36$$

$$\underline{x^2 - 12x + 36} - 3 - 36 = 0$$

perfect square

$$(x - 6)^2 - 3 - 36 = 0$$

$$(x - 6)^2 - 39 = 0$$

$$\begin{aligned}(x - 6)^2 - 39 &= 0 \\ \underline{\quad\quad\quad +39 \quad +39} & \\ (x - 6)^2 &= 39 \\ \sqrt{(x - 6)^2} &= \sqrt{39} \\ x - 6 &= \pm\sqrt{39} \\ \underline{\quad\quad\quad +6 \quad\quad\quad +6} & \\ x &= \pm\sqrt{39} + 6\end{aligned}$$

when  $a \neq 1$

You will two additional steps...

### Steps

1) Put equation in standard form ( $ax^2 + bx + c$ )

### ADDITIONAL STEP

Factor out the  $a$  from the  $x^2$ - and  $x$ -terms.

### Example

$$3x^2 + 21x = -4$$

$$3x^2 + 21x + 4 = 0$$

$$3(x^2 + 7x) + 4 = 0$$

You will now complete the square as before inside the parentheses.

2) Divide coefficient of the  $x$ -term by 2 and square it.  
This is the number that actually completes the square.

$$\left(\frac{7}{2}\right)^2 = \frac{49}{4}$$

3) Add AND subtract the number you found in step 2.  
Always add first.

$$3\left(x^2 + 7x + \frac{49}{4} - \frac{49}{4}\right) + 4 = 0$$

### ADDITIONAL STEP

Distribute  $a$  to the subtracted term to remove it from the parentheses. You want to have ONLY the perfect square inside the parentheses.

$$3\left(x^2 + 7x + \frac{49}{4}\right) - 3\left(\frac{49}{4}\right) + 4 = 0$$

perfect square

4) Rewrite the perfect square as a squared term.

$$3\left(x + \frac{7}{2}\right)^2 - 3\left(\frac{49}{4}\right) + 4 = 0$$

5) Simplify.

$$3\left(x + \frac{7}{2}\right)^2 - \left(\frac{147}{4}\right) + 4 = 0$$
$$3\left(x + \frac{7}{2}\right)^2 - \left(\frac{147}{4}\right) + \frac{16}{4} = 0$$
$$3\left(x + \frac{7}{2}\right)^2 - \frac{131}{4} = 0$$

6) Solve for  $x$ .

$$3\left(x + \frac{7}{2}\right)^2 - \frac{131}{4} = 0$$
$$3\left(x + \frac{7}{2}\right)^2 = \frac{131}{4}$$
$$\left(x + \frac{7}{2}\right)^2 = \frac{131}{12}$$
$$x + \frac{7}{2} = \pm \sqrt{\frac{131}{12}}$$
$$x = -\frac{7}{2} \pm \sqrt{\frac{131}{12}}$$