(b) Factoring, we find $y = (x + 2)(x - 2)(x - 2)$. There is a double zero at $x = 2$, a zero at $x = -2$, and no other zeros (since it is a cubic).

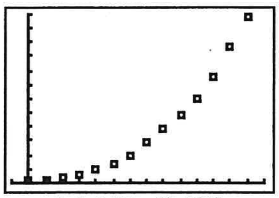
(c) Same visually as graph in (a).

(d) $b^2 - 4ac$ is the discriminant. In this case, $b^2 - 4ac = (-4)^2 - 4(1)(4.01) = -0.04$, which is negative. So the only real zero of the product $y = (x + 2)(x^2 - 4x + 4.01)$ is at $x = -2$.

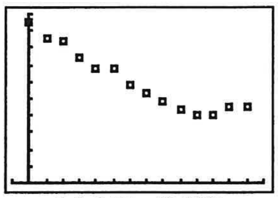
(e) Same visually as the graph in (a).

(f) $b^2 - 4ac = (-4)^2 - 4(1)(3.99) = 0.04$, which is positive. The discriminant will provide two real zeros of the quadratic, and $(x + 2)$ provides the third. A cubic equation can have no more than three real roots.

63. (a)



[–1, 14] by [0, 120]



[–1, 14] by [0, 100]

(b) Solving at $t = 0$: $1.6 = a \cdot 0 + b$

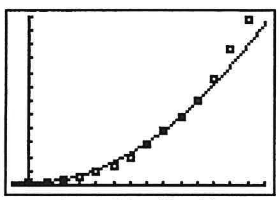
$1.6 = b$

Solving at $t = 9$: $48.7 = a \cdot 81 + 1.6$

$$a = \frac{48.7 - 1.6}{81} = \frac{47.1}{81} \approx 0.581$$

$y = 0.581x^2 + 1.6$

(c)



[–1, 14] by [0, 120]

This is a fairly good fit.

(d) Solving at $t = 1$: $85.52 = m \cdot 1 + b$

$85.52 = m + b$

Solving at $t = 12$: $45.15 = m \cdot 12 + b$

There are two equations with two unknowns. Solve for m.

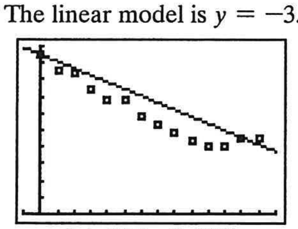
$$\begin{aligned} 85.52 &= m + b \\ -\,(45.15 &= 12m + b) \\ \hline 40.37 &= -11m \\ m &= -3.67 \end{aligned}$$

Find b.

$$\begin{aligned} 85.52 &= -3.67 + b \\ b &= 89.19 \end{aligned}$$

The linear model is $y = -3.67x + 89.19$.

(e)

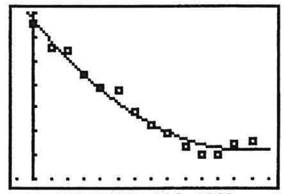


[–1, 14] by [0, 100]

The fit is fair, but could be better.

(f) Yes. The points seem to be curved beneath the line and might be better approximated by a parabola.

(g) $y = 0.342x^2 - 8.64x + 96.82$



[–1, 14] by [30, 100]

This fits better than the line.

64. One possible answer: The number of cell phone users is increasing dramatically (as the quadratic model shows), and the average monthly bill is going down as more people share the industry cost. The model shows that the number of users will continue to rise, although the quadratic model cannot hold up indefinitely. The decline in the average monthly bill tapers off as the graph approaches a vertex, and there are slight price increases in the last three years. But again, the quadratic model cannot apply indefinitely.

■ Section 1.2 Functions and Their Properties

Exploration 1

1. From left to right, the tables are (c) constant, (b) decreasing, and (a) increasing.

2.

X moves from	ΔX	ΔY1	X moves from	ΔX	ΔY2	X moves from	ΔX	ΔY3
–2 to –1	1	0	–2 to –1	1	–2	–2 to –1	1	2
–1 to 0	1	0	–1 to 0	1	–1	–1 to 0	1	2
0 to 1	1	0	0 to 1	1	–2	0 to 1	1	2
1 to 3	2	0	1 to 3	2	–4	1 to 3	2	3
3 to 7	4	0	3 to 7	4	–6	3 to 7	4	6

3. For an increasing function, ΔY/ΔX is positive. For a decreasing function, ΔY/ΔX is negative. For a constant function, ΔY/ΔX is 0.

4. For lines, ΔY/ΔX is the slope. Lines with positive slope are increasing, lines with negative slopes are decreasing, and lines with 0 slope are constant, so this supports our answers to part 3.

Quick Review 1.2

1. $x^2 - 16 = 0$

   $x^2 = 16$

   $x = \pm 4$

2. $9 - x^2 = 0$

   $9 = x^2$

   $\pm 3 = x$

3. $x - 10 < 0$

   $x < 10$

4. $5 - x \le 0$

   $-x \le -5$

   $x \ge 5$

5. As we have seen, the denominator of a function cannot be zero.
We need $x - 16 = 0$
$x = 16$

6. We need $x^2 - 16 = 0$
$x^2 = 16$
$x = \pm 4$

7. We need $x - 16 < 0$
$x < 16$

8. We need $x^2 - 1 = 0$
$x^2 = 1$
$x = \pm 1$

9. We need $3 - x \leq 0$ and $x + 2 < 0$
$3 \leq x$ $\quad x < -2$
$x < -2$ and $x \geq 3$

10. We need $x^2 - 4 = 0$
$x^2 = 4$
$x = \pm 2$

Section 1.2 Exercises

1. Yes, $y = \sqrt{x - 4}$ is a function of x, because when a number is substituted for x, there is at most one value produced for $\sqrt{x - 4}$.

2. No, $y = x^2 \pm 3$ is not a function of x, because when a number is substituted for x, y can be either 3 more or 3 less than x^2.

3. No, $x = 2y^2$ does not determine y as a function of x, because when a positive number is substituted for x, y can be either $\sqrt{\frac{x}{2}}$ or $-\sqrt{\frac{x}{2}}$.

4. Yes, $x = 12 - y$ determines y as a function of x, because when a number is substituted for x, there is exactly one number y which, when subtracted from 12, produces x.

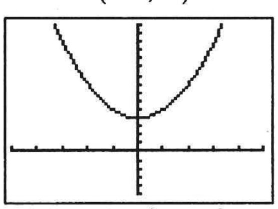
5. Yes

6. No

7. No

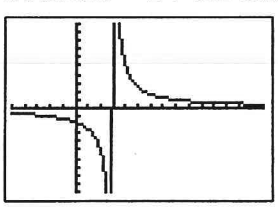
8. Yes

9. We need $x^2 + 4 \geq 0$; this is true for all real x.
Domain: $(-\infty, \infty)$.



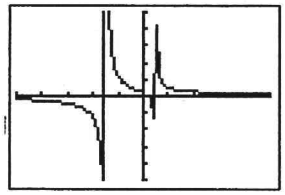
$[-5, 5]$ by $[-5, 15]$

10. We need $x - 3 \neq 0$. Domain: $(-\infty, 3) \cup (3, \infty)$.



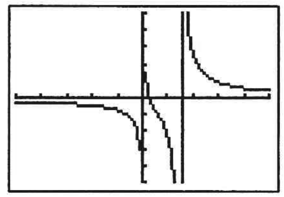
$[-5, 15]$ by $[-10, 10]$

11. We need $x + 3 \neq 0$ and $x - 1 \neq 0$. Domain: $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$.



$[-10, 10]$ by $[-10, 10]$

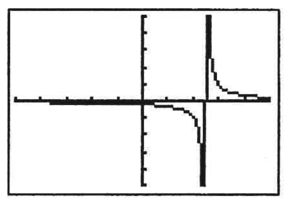
12. We need $x \neq 0$ and $x - 3 \neq 0$. Domain: $(-\infty, 0) \cup (0, 3) \cup (3, \infty)$.



$[-10, 10]$ by $[-10, 10]$

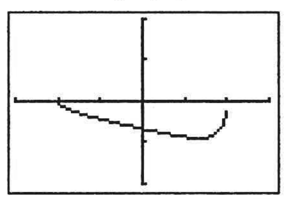
13. We notice that $g(x) = \dfrac{x}{x^2 - 5x} = \dfrac{x}{x(x - 5)}$.

As a result, $x - 5 \neq 0$ and $x \neq 0$.
Domain: $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$.



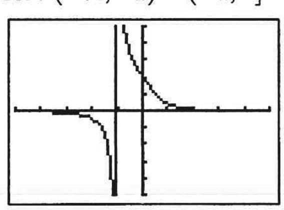
$[-10, 10]$ by $[-5, 5]$

14. We need $x - 3 \neq 0$ and $4 - x^2 \geq 0$. This means $x \neq 3$ and $x^2 \leq 4$; the latter implies that $-2 \leq x \leq 2$, so the domain is $[-2, 2]$.



$[-3, 3]$ by $[-2, 2]$

15. We need $x + 1 \neq 0$, $x^2 + 1 \neq 0$, and $4 - x \geq 0$. The first requirement means $x \neq -1$, the second is true for all x, and the last means $x \leq 4$. The domain is therefore $(-\infty, -1) \cup (-1, 4]$.

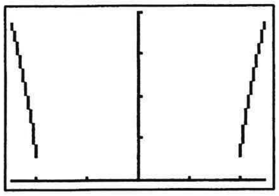


$[-5, 5]$ by $[-5, 5]$

16. We need

$$x^4 - 16x^2 \geq 0$$
$$x^2(x^2 - 16) \geq 0$$
$$x^2 = 0 \quad \text{or} \quad x^2 - 16 \geq 0$$
$$x^2 \geq 16$$
$$x = 0 \quad \text{or} \quad x \geq 4, \; x \leq -4$$

Domain: $(-\infty, -4] \cup \{0\} \cup [4, \infty)$

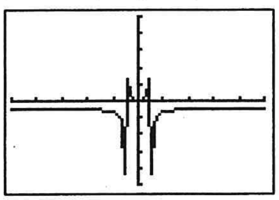


$[-5, 5]$ by $[0, 16]$

17. $f(x) = 10 - x^2$ can take on any negative value. Because x^2 is nonnegative, $f(x)$ cannot be greater than 10. The range is $(-\infty, 10]$.

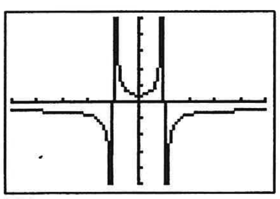
18. $g(x) = 5 + \sqrt{4 - x}$ can take on any value ≥ 5, but because $\sqrt{4 - x}$ is nonnegative, $g(x)$ cannot be less than 5. The range is $[5, \infty)$.

19. The range of a function is most simply found by graphing it. As our graph shows, the range of $f(x)$ is $(-\infty, -1) \cup [0, \infty)$.



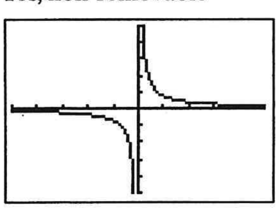
$[-10, 10]$ by $[-10, 10]$

20. As our graph illustrates, the range of $g(x)$ is $(-\infty, -1) \cup [\frac{3}{4}, \infty)$.



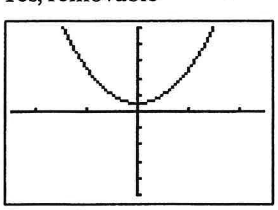
$[-10, 10]$ by $[-10, 10]$

21. Yes, non-removable



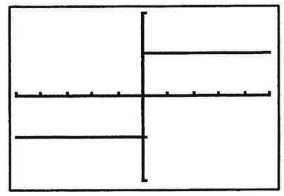
$[-10, 10]$ by $[-10, 10]$

22. Yes, removable



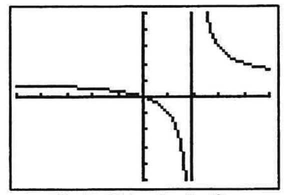
$[-5, 5]$ by $[-10, 10]$

23. Yes, non-removable



$[-10, 10]$ by $[-2, 2]$

24. Yes, non-removable



$[-5, 5]$ by $[-5, 5]$

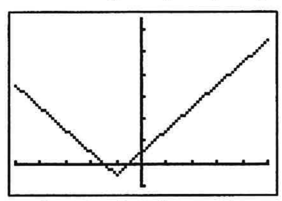
25. Local maxima at $(-1, 4)$ and $(5, 5)$, local minimum at $(2, 2)$. The function increases on $(-\infty, -1]$, decreases on $[-1, 2]$, increases on $[2, 5]$, and decreases on $[5, \infty)$.

26. Local minimum at $(1, 2)$, $(3, 3)$ is neither, and $(5, 7)$ is a local maximum. The function decreases on $(-\infty, 1]$, increases on $[1, 5]$, and decreases on $[5, \infty)$.

27. $(-1, 3)$ and $(3, 3)$ are neither. $(1, 5)$ is a local maximum, and $(5, 1)$ is a local minimum. The function increases on $(-\infty, 1]$, decreases on $[1, 5]$, and increases on $[5, \infty)$.

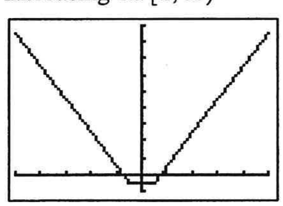
28. $(-1, 1)$ and $(3, 1)$ are local minima, while $(1, 6)$ and $(5, 4)$ are local maxima. The function decreases on $(-\infty, -1]$, increases on $[-1, 1]$, decreases on $(1, 3]$, increases on $[3, 5]$, and decreases on $[5, \infty)$.

29. Decreasing on $(-\infty, -2]$; increasing on $[-2, \infty)$



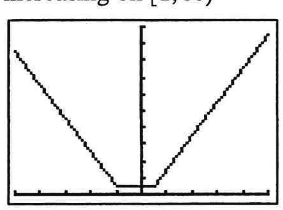
$[-10, 10]$ by $[-2, 18]$

30. Decreasing on $(-\infty, -1]$; constant on $[-1, 1]$; increasing on $[1, \infty)$



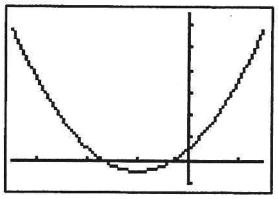
$[-10, 10]$ by $[-2, 18]$

31. Decreasing on $(-\infty, -2]$; constant on $[-2, 1]$; increasing on $[1, \infty)$



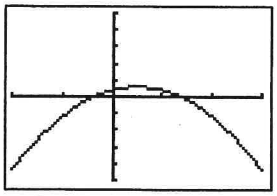
$[-10, 10]$ by $[0, 20]$

32. Decreasing on $(-\infty, -2]$; increasing on $[-2, \infty)$



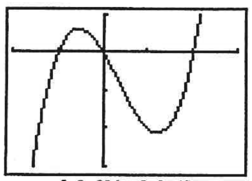
$[-7, 3]$ by $[-2, 13]$

33. Increasing on $(-\infty, 1]$; decreasing on $[1, \infty)$



$[-4, 6]$ by $[-25, 25]$

34. Increasing on $(-\infty, -0.5]$; decreasing on $[-0.5, 1.2]$, increasing on $[1.2, \infty)$. The middle values are approximate —they are actually at about -0.549 and 1.215. The values given are what might be observed on the decimal window.



$[-2, 3]$ by $[-3, 1]$

35. Constant functions are always bounded.

36.
$$x^2 > 0$$
$$-x^2 < 0$$
$$2 - x^2 < 2$$
y is bounded above by $y = 2$.

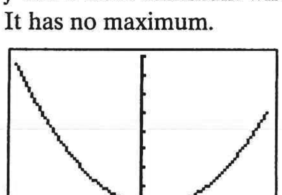
37. $2^x > 0$ for all x, so y is bounded below by $y = 0$.

38. $2^{-x} = \dfrac{1}{2^x} > 0$ for all x, so y is bounded below by $y = 0$.

39. Since $y = \sqrt{1 - x^2}$ is always positive, we know that $y \geq 0$ for all x. We must also check for an upper bound:
$$x^2 > 0$$
$$-x^2 < 0$$
$$1 - x^2 < 1$$
$$\sqrt{1 - x^2} < \sqrt{1}$$
$$\sqrt{1 - x^2} < 1$$
Thus, y is bounded.

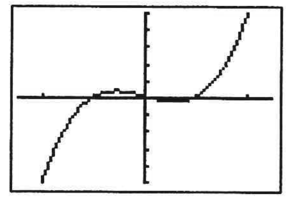
40. There are no restrictions on either x or x^3, so y is not bounded.

41. f has a local minimum when $x = 0.5$, where $y = 3.75$. It has no maximum.



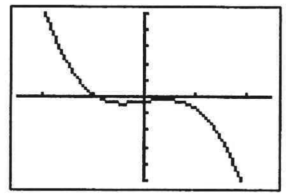
$[-5, 5]$ by $[0, 36]$

42. Local maximum: $y \approx 4.08$ at $x \approx -1.15$.
Local minimum: $y \approx -2.08$ at $x \approx 1.15$.



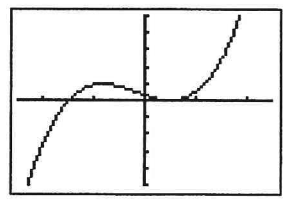
$[-5, 5]$ by $[-50, 50]$

43. Local minimum: $y \approx -4.09$ at $x \approx -0.82$.
Local maximum: $y \approx -1.91$ at $x \approx 0.82$.



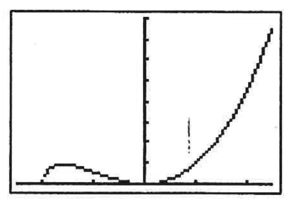
$[-5, 5]$ by $[-50, 50]$

44. Local maximum: $y \approx 9.48$ at $x \approx -1.67$.
Local minimum: $y = 0$ when $x = 1$.



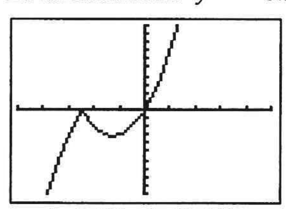
$[-5, 5]$ by $[-50, 50]$

45. Local maximum: $y \approx 9.16$ at $x \approx -3.20$.
Local minima: $y = 0$ at $x = 0$ and $y = 0$ at $x = -4$.



$[-5, 5]$ by $[0, 80]$

46. Local maximum: $y = 0$ at $x = -2.5$.
Local minimum: $y \approx -3.13$ at $x = -1.25$.



$[-5, 5]$ by $[-10, 10]$

47. Even: $f(-x) = 2(-x)^4 = 2x^4 = f(x)$

48. Odd: $g(-x) = (-x)^3 = -x^3 = -g(x)$

49. Even: $f(-x) = \sqrt{(-x)^2 + 2} = \sqrt{x^2 + 2} = f(x)$

50. Even: $g(-x) = \dfrac{3}{1 + (-x)^2} = \dfrac{3}{1 + x^2} = g(x)$

51. Neither: $f(-x) = -(-x)^2 + 0.03(-x) + 5 = -x^2 - 0.03x + 5$, which is neither $f(x)$ nor $-f(x)$.

52. Neither: $f(-x) = (-x)^3 + 0.04(-x)^2 + 3 = -x^3 + 0.04x^2 + 3$, which is neither $f(x)$ nor $-f(x)$.