

$$1. \frac{1}{\sin^2 x} - \frac{1}{\tan^2 x} = \csc^2 x - \cot^2 x = \boxed{1}$$

$$2. \frac{1 + \cos x \tan x \csc x}{\csc x} = \frac{1 + \cos x \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}}{\csc x} = \frac{1 + \frac{\cos^2 x}{\sin^2 x}}{\csc x} =$$

$$\frac{1 + \tan^2 x}{\csc x} = \frac{\csc^2 x}{\csc x} = \boxed{\csc x}$$

$$\frac{c^2 + s^2}{s^2} = \frac{1}{s^2}$$

$$3. \frac{\sin^4 x - \sin^2 x}{\sec x} = \frac{\sin^2 x (\sin^2 x - 1)}{\sec x} = \frac{\sin^2 x \cdot \cos^2 x}{\frac{1}{\cos x}} = \boxed{-\sin^2 x \cos^3 x}$$

$$4. \frac{-\tan^2 x - 1}{\sec x} = \frac{-1(\tan^2 x + 1)}{\sec x} = \frac{-1 \sec^2 x}{\sec x} = \boxed{-\sec x}$$

$$5. \frac{\tan^3 x - \sec^2 x \tan x}{\cot(-x)} = \frac{\tan x (\tan^2 x - \sec^2 x)}{-\cot x} = -\tan x \cdot -\tan x = \boxed{\tan^2 x}$$

$$6. \sin x + \frac{\cos^2 x}{\sin x} = \frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x} = \frac{1}{\sin x} = \boxed{\csc x}$$

$$7. \frac{1}{\sin^3 x} + \frac{\cot^2 x}{\sin(-x)} = \frac{1}{\sin^3 x} - \frac{\frac{\cos^2 x}{\sin^2 x}}{\sin x} = \frac{1}{\sin^3 x} - \frac{\cos^2 x}{\sin^3 x} = \frac{1 - \cos^2 x}{\sin^3 x} = \frac{\sin^2 x}{\sin^3 x} = \frac{1}{\sin x} = \boxed{\csc x}$$

$$8. \sin(-x) \cot(-x) = -\sin x \cdot -\cot x = \sin x \cdot \frac{\cos x}{\sin x} = \boxed{\cos x}$$

$$9. \frac{\sin x}{\cos(-x)} + \frac{\sin(-x)}{\cos x} = \frac{\sin x}{\cos x} - \frac{\sin x}{\cos x} = \boxed{0}$$

$$10. \frac{1}{\cos(\frac{\pi}{2}-\theta)} = \frac{1}{\sin\theta} = \boxed{\csc\theta}$$

$$11. \frac{\tan\theta}{\csc(\theta-\frac{\pi}{2})} = \frac{\tan\theta}{\csc(-(\frac{\pi}{2}-\theta))} = \frac{\tan\theta}{-\csc(\frac{\pi}{2}-\theta)} = \frac{\tan\theta}{-\sec\theta} = \frac{\sin\theta}{\cos\theta} \cdot \frac{-\cos\theta}{1}$$

$$= \boxed{-\sin\theta}$$

$$12. \frac{1}{\cos^2 x} - \frac{1}{\cot^2 x} = \sec^2 x - \tan^2 x = \boxed{1}$$

$$\frac{c^2+s^2}{c^2} - \frac{1}{\frac{c^2}{s^2}} = 1$$

$$13. \cos\theta (\sec\theta - \cos\theta) = \cos\theta \sec\theta - \cos^2\theta = \cos\theta \left(\frac{1}{\cos\theta}\right) - \cos^2\theta$$

$$= 1 - \cos^2\theta = \boxed{\sin^2\theta}$$

$$14. (\csc x - 1)(\csc x + 1) = \csc^2 x - 1 = \boxed{\cot^2 x}$$

$$\frac{c^2+s^2}{s^2} - \frac{1}{\frac{c^2}{s^2}} = 1$$

$$15. \frac{\sec^2\theta - \tan^2\theta}{\csc(\frac{\pi}{2}-\theta)} = \frac{1}{\sec\theta} = \boxed{\cos\theta}$$

$$\frac{c^2+s^2}{c^2} - \frac{1}{\frac{c^2}{s^2}} = 1$$

$$16. \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = \sin x \left(\frac{\sin x}{1}\right) + \cos x \left(\frac{\cos x}{1}\right) = \sin^2 x + \cos^2 x = \boxed{1}$$

$$17. \cos\theta = ? \quad \sin\theta = ? \quad \tan\theta = \frac{1}{2} \quad \sin\theta > 0 \quad \text{use } \tan^2\theta + 1 = \sec^2\theta$$

$$\left(\frac{1}{2}\right)^2 + 1 = \sec^2\theta = \frac{1}{4} + 1 = \frac{5}{4} \quad \sec\theta = \frac{\sqrt{5}}{2} \quad \boxed{\cos\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}}$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(\frac{2}{\sqrt{5}}\right)^2 + \sin^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \frac{4}{5} = \frac{1}{5} \Rightarrow \boxed{\sin\theta = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}}$$

$$18. \tan \theta = ? \quad \sec \theta = ? \quad \sin \theta = \frac{3}{4} \quad \cos \theta < 0$$

$$\tan \theta < 0 + \sec \theta < 0$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\left(\frac{3}{4}\right)^2 + \cos^2 \theta = 1 \Rightarrow \cos^2 \theta = 1 - \frac{9}{16} = \frac{7}{16} \Rightarrow \cos \theta = -\frac{\sqrt{7}}{4} \Rightarrow \sec \theta = -\frac{4}{\sqrt{7}}$$

$$\boxed{\sec \theta = -\frac{4\sqrt{7}}{7}}$$

$$\tan^2 \theta + 1 = \sec^2 \theta \quad \tan^2 \theta + 1 = \left(-\frac{4}{\sqrt{7}}\right)^2 = \frac{16}{7}$$

$$\tan^2 \theta = \frac{16}{7} - 1 = \frac{9}{7}$$

$$\boxed{\tan \theta = -\frac{3}{\sqrt{7}} = -\frac{3\sqrt{7}}{7}}$$

$$19. \csc \theta = ? \quad \tan \theta = ? \quad \sec \theta = \frac{-4\sqrt{5}}{5} = -\frac{4}{\sqrt{5}} \quad \sin \theta > 0$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\csc \theta > 0 + \tan \theta < 0$$

$$\tan^2 \theta + 1 = \left(-\frac{4}{\sqrt{5}}\right)^2 = \frac{16}{5} \Rightarrow \tan^2 \theta = \frac{16}{5} - 1 = \frac{11}{5} \Rightarrow \boxed{\tan \theta = -\sqrt{\frac{11}{5}}}$$

$$\cos \theta = -\frac{\sqrt{5}}{4} \Rightarrow \left(-\frac{\sqrt{5}}{4}\right)^2 + \sin^2 \theta = 1 \Rightarrow \frac{5}{16} + \sin^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = 1 - \frac{5}{16} = \frac{11}{16} \Rightarrow \sin \theta = \frac{\sqrt{11}}{4} \Rightarrow \boxed{\csc \theta = \frac{4}{\sqrt{11}} = \frac{4\sqrt{11}}{11}}$$

$$20. \frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 4$$

$$\frac{(1 + \sin x)^2 + \cos^2 x}{(\cos x)(1 + \sin x)} = 4$$

$$\frac{1 + 2\sin x + \sin^2 x + \cos^2 x}{\cos x (1 + \sin x)} = 4$$

$$\frac{2 + 2\sin x}{\cos x (1 + \sin x)} = 4$$

$$\frac{2(1 + \sin x)}{\cos x (1 + \sin x)} = 4$$

$$\cos x = \frac{1}{2} \quad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$21. \cot x \cos^2 x = 2 \cot x$$

$$\cot x \cos^2 x - 2 \cot x = 0$$

$$\cot x (\cos^2 x - 2) = 0$$

$$\cot x = 0 \quad \cos^2 x - 2 = 0$$

$$\downarrow \quad \cos x = \sqrt{2} \quad \text{Not possible}$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$22. 2\sin^2\theta = 3\cos\theta$$

$$2(1 - \cos^2\theta) = 3\cos\theta$$

$$2 - 2\cos^2\theta = 3\cos\theta$$

$$2\cos^2\theta + 3\cos\theta - 2 = 0$$

$$(2\cos\theta - 1)(\cos\theta + 2) = 0$$

$$\cos\theta = \frac{1}{2}$$

$$\cos\theta = -2$$

Not possible

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$23. \tan x = 3\sin x$$

$$\frac{\sin x}{\cos x} = 3\sin x$$

$$3\sin x - \frac{\sin x}{\cos x} = 0$$

$$\sin x (3 - \sec x) = 0$$

$$\sin x = 0$$

$$3 - \sec x = 0$$

$$\sec x = 3$$

$$\cos x = \frac{1}{3}$$

$$x = \cos^{-1}\left(\frac{1}{3}\right)$$

$$x = 1.239$$

$$5.05$$

$$x = 0, \pi, 2\pi$$

$$24. \sec\theta = 2\cos\theta$$

$$\frac{1}{\cos\theta} = 2\cos\theta$$

$$1 = 2\cos^2\theta$$

$$\frac{1}{2} = \cos^2\theta$$

$$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4}, \frac{7\pi}{4}$$

$$25. \frac{4}{\sec^2 x} + 3\cos x = 2\cot x \tan x$$

$$4\cos^2 x + 3\cos x = \frac{2\cos x \tan x}{\tan x \cos x}$$

$$4\cos^2 x + 3\cos x = 2$$

$$4\cos^2 x + 3\cos x - 2 = 0$$

$$(2\cos x - 1)(2\cos x + 2) = 0$$

$$\cos x = \frac{1}{2} \quad \cos x = -1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \quad x = \pi$$

$$26. 4\sin^2 x + 3\cos x = 2$$

$$26. \underline{4\sin x \cos x} + \underline{2\sin x} - 2\cos x - 1 = 0$$

$$2\sin x (2\cos x + 1) - (2\cos x + 1) = 0$$

$$(2\sin x - 1)(2\cos x + 1) = 0$$

$$\sin x = \frac{1}{2} \quad \cos x = -\frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$27. \cos^2 \theta = -6\sin \theta$$

$$1 - \sin^2 \theta = -6\sin \theta$$

$$\sin^2 \theta - 6\sin \theta - 1 = 0$$

Doesn't factor \Rightarrow Use Quad Formula

$$a=1 \quad b=-6 \quad c=-1$$

$$\sin \theta = \frac{6 \pm \sqrt{36 - 4(-1)}}{2}$$

$$\sin \theta = \frac{6 \pm \sqrt{40}}{2}$$

$$\sin \theta = \frac{6 \pm 2\sqrt{10}}{2}$$

$$\sin \theta = 3 \pm \sqrt{10}$$

$$\sin \theta = 3 + \sqrt{10} \quad \text{Not possible}$$

$$\sin \theta = 3 - \sqrt{10} = -.1623$$

$$\theta = 6.120, 3.305$$