

$$1. \tan \theta = 6/5 \quad \cos \theta < 0 \quad a=6 \quad b=5 \quad c=\sqrt{61}$$

$$\begin{aligned} \tan \theta &= 6/5 & \cos \theta &= -\frac{5\sqrt{61}}{61} & \sin \theta &= \frac{-6\sqrt{61}}{61} \\ \cot \theta &= 5/6 & \sec \theta &= -\frac{\sqrt{61}}{5} & \csc \theta &= -\frac{\sqrt{61}}{6} \end{aligned}$$

$$2. \csc^2 \beta (1 - \cos^2 \beta) = \frac{1}{\sin^2 \beta} \cdot \sin^2 \beta = 1$$

$$\begin{aligned} 3. \frac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x} &= \frac{(\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)}{\sec^2 x + \tan^2 x} = \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} \\ &= \frac{1 - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = 1 \end{aligned}$$

$$4. \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} = \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta} = \csc \theta \sec \theta$$

$$\begin{aligned} 5. \tan \theta &= -\sqrt{\sec^2 \theta - 1} & c^2 + s^2 &= 1 \\ \tan \theta &= -\sqrt{\tan^2 \theta} & \frac{c^2}{c^2} + \frac{s^2}{c^2} &= \frac{1}{c^2} \\ \tan \theta &= -|\tan \theta| & 1 + \tan^2 x &= \sec^2 x \end{aligned}$$

$\theta = 0, 2^{\text{nd}}$ quad, $\pi, 4^{\text{th}}$ quad.

$$6. \cos x + \sin x \tan x = \sec x$$

$$\cos x + \sin x \cdot \frac{\sin x}{\cos x}$$

$$\frac{\cos^2 x + \sin^2 x}{\cos x}$$

$$\frac{1}{\cos x}$$

$$\sec x = \sec x \quad \checkmark$$

$$7. \sin \theta \sec \theta = \tan \theta$$

$$\sin \theta \cdot \frac{1}{\cos \theta}$$

$$\frac{\sin \theta}{\cos \theta}$$

$$\tan \theta = \tan \theta \quad \checkmark$$

$$8. \sec^2 x \tan^2 x + \sec^2 x = \sec^4 x$$

$$\sec^2 x (\tan^2 x + 1)$$

$$\sec^2 x \cdot \sec^2 x$$

$$\sec^4 x = \sec^4 x \quad \checkmark$$

$$9. \frac{\csc x + \sec x}{\sin x + \cos x} = \cot x + \tan x$$

$$\frac{1}{\sin x} + \frac{1}{\cos x} \cdot \frac{\cos x + \sin x}{\sin x + \cos x}$$

$$\frac{\cos x + \sin x}{\sin x \cos x}$$

$$\frac{\cos x + \sin x}{\sin x \cos x}$$

$$\frac{\sin x + \cos x}{\sin x \cos x}$$

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$$\frac{\cos x + \sin x}{\sin x \cos x}$$

$$\frac{\cos x + \sin x}{\sin x \cos x}$$

\Rightarrow

$$\frac{1}{\sin x \cos x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{\sin x}{\sin x \cos x} + \frac{\cos x}{\sin x \cos x} =$$

$$\tan x + \cot x = \cot x + \tan x \quad \checkmark$$

$$10. \tan(x + \pi/2) = -\cot x \quad \checkmark$$

$$\frac{\sin(x + \pi/2)}{\cos(x + \pi/2)} = \frac{\sin x \cos \pi/2 + \cos x \sin \pi/2}{\cos x \cos \pi/2 - \sin x \sin \pi/2} = \frac{\cos x}{-\sin x} = -\cot x \quad \checkmark$$

$$11. \sin(x + n\pi) = (-1)^n \sin x \quad \checkmark$$

$$\frac{\sin x \cos n\pi + \cos x \sin n\pi}{\cos x \cos n\pi + \sin x \sin n\pi} = \frac{\sin x \cos n\pi}{\cos x \cos n\pi} = \tan x \cdot \frac{\cos n\pi}{\cos n\pi} = \tan x \cdot 1 = \tan x$$

Note: $\cos n\pi = \pm 1 = (-1)^n$

So, $\sin(x + n\pi) = (-1)^n \sin x \quad \checkmark$

$$12. (\sin x + \cos x)^2 = 1 + \sin 2x$$

$$\sin^2 x + \cos^2 x + 2 \sin x \cos x = 1 + 2 \sin x \cos x = 1 + \sin 2x \quad \checkmark$$

$$14. \frac{\sin 4\theta}{1 + \cos 4\theta} = \tan(2\theta) \quad \checkmark$$

$$17. \tan^2 x + \tan x = 0; [0, 2\pi)$$

$$\tan x (\tan x + 1) = 0$$

$$\tan x = 0 \quad \tan x = -1$$

$$x = 0, \pi \quad x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

$$18. \sin 2x - \cos x = 0$$

$$2 \sin x \cos x - \cos x = 0$$

$$\cos x (2 \sin x - 1) = 0$$

$$\cos x = 0 \quad \sin x = 1/2$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$19. 4 \cos^2 x - 3 = 0$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm \frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$20. \csc^2 x - \csc x - 2 = 0$$

$$(\csc x - 2)(\csc x + 1) = 0$$

$$\csc x = 2 \quad \csc x = -1$$

$$\sin x = 1/2 \quad \sin x = -1$$

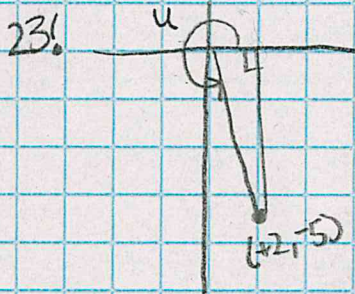
$$x = \frac{\pi}{6}, \frac{5\pi}{6} \quad x = \frac{3\pi}{2}$$

$$22. \cos 105^\circ = \cos (135^\circ - 30^\circ)$$

$$= \cos 135^\circ \cos 30^\circ - \sin 135^\circ \sin 30^\circ$$

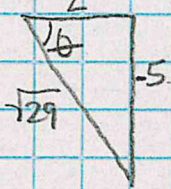
$$= -\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{-\sqrt{6} - \sqrt{2}}{4}$$



$$\sin u = \frac{-5}{\sqrt{29}} = \frac{-5\sqrt{29}}{29}$$

$$\cos u = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$$



Note: $2u$ is in Q3 \Rightarrow s & c are -
tan is +

$$\sin 2u = 2 \sin u \cos u = 2 \left(\frac{-5\sqrt{29}}{29} \right) \left(\frac{2}{\sqrt{29}} \right) = \frac{-20}{\sqrt{29}} = \frac{-20\sqrt{29}}{29}$$

$$\cos 2u = \cos^2 u - \sin^2 u = \left(\frac{2}{\sqrt{29}} \right)^2 - \left(\frac{-5}{\sqrt{29}} \right)^2 = \frac{4 - 25}{29}$$

$$= \frac{-21}{29}$$

$$\tan 2u = \frac{2 \cos u \sin u}{\cos^2 u - \sin^2 u} = \frac{\sin 2u}{\cos 2u} = \frac{\frac{-20\sqrt{29}}{29}}{\frac{-21}{29}} = \frac{20\sqrt{29}}{21}$$

~~23!~~

