

Pre-Calculus  
Mid-Chapter Review

Name: *Key*  
Date:

Period:

1. Convert between degrees and radians or vice versa.

a)  $324^\circ$

b)  $65^\circ$

c)  $\frac{7\pi}{9}$

d)  $-\frac{8\pi}{15}$

2. Evaluate without a calculator.

a)  $\tan \frac{2\pi}{3}$

b)  $\sec \frac{3\pi}{4}$

c)  $\sin -\frac{7\pi}{4}$

d)  $\cot \frac{5\pi}{6}$

3. Solve. Pay attention to the given domain.

a)  $\sin x = 0; 0 \leq x < 2\pi$

b)  $3 - 4\cos^2 x = 0; 0 \leq x < \pi$

c)  $1 - 4\sin^2 x = -1; \pi \leq x < 2\pi$

d)  $3 + 2\cos x = 4; \pi \leq x < 2\pi$

4. Solve.

a)  $\sin(3x) = -1$

b)  $6\cos\left(x - \frac{\pi}{4}\right) = 3$

5. If  $\tan x = 2$  and  $\cos x < 0$ , find the other 5 trig values.

$\sin x =$

$\cos x =$

$\tan x =$

$\sec x =$

$\cot x =$

6. If  $\sec x = \frac{3}{2}$  and  $\frac{3\pi}{2} < x < 2\pi$ , find the other 5 trig values.

$\sin x =$

$\cos x =$

$\tan x =$

$\sec x =$

$\cot x =$

7. A six-meter-long ladder leans against a building. If the ladder makes an angle of  $60^\circ$  with the ground, how far up the wall does the ladder reach? How far from the wall is the base of the ladder? Round your answers to two decimal places, as needed.

8. Suppose you have been assigned to measure the height of the local water tower. Climbing makes you dizzy, so you decide to do the whole job at ground level. From a point 47.3 meters from the base of the water tower, you find that you must look up at an angle of  $53^\circ$  to see the top of the tower. How tall is the tower? Draw the triangle.

9. An observer 5.2 kilometers from the launch pad observes a missile ascending. At a particular, the angle of elevation is  $37.6^\circ$ . How high is the missile?

10. For each function below, identify amplitude, frequency, phase shift, axis, and period. Then graph each function.

$y = 1 + 4\cot\left(\frac{x}{2} + \frac{\pi}{6}\right)$

$f(x) = 3\sec\left(2x + \frac{3\pi}{2}\right) + 2$

$f(x) = -2 + 3\csc\left(\frac{x}{4} + \frac{3\pi}{8}\right)$

$y = \frac{1}{4}\tan\left(4x + \frac{4\pi}{3}\right) - 1$

$$1. a) 324^\circ \cdot \frac{\pi}{180^\circ} = \frac{9\pi}{5} \quad b) 65^\circ \cdot \frac{\pi}{180^\circ} = \frac{13\pi}{36}$$

$$c) \frac{7\pi}{9} \cdot \frac{180^\circ}{\pi} = 140^\circ \quad d) -\frac{8\pi}{15} \cdot \frac{180^\circ}{\pi} = -96^\circ$$

$$2. a) \tan \frac{2\pi}{3} = \frac{\sin \frac{2\pi}{3}}{\cos \frac{2\pi}{3}} = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$$

$$b) \sec \frac{3\pi}{4} = \frac{1}{\cos \frac{3\pi}{4}} = \frac{1}{-\sqrt{2}/2} = \frac{-2}{\sqrt{2}} = -\sqrt{2}$$

$$c) \sin \frac{-7\pi}{4} = \frac{\sqrt{2}}{2} \quad d) \cot \frac{5\pi}{6} = \frac{\cos \frac{5\pi}{6}}{\sin \frac{5\pi}{6}} = \frac{-\frac{\sqrt{3}}{2}}{1/2} = -\sqrt{3}$$

$\oplus$

$$3. a) \sin x = 0; 0 \leq x < 2\pi \quad x = 0, \pi$$

$x = 0, \pi, 2\pi$

$$b) 3 - 4\cos^2 x = 0; 0 \leq x < \pi$$

$$\cos^2 x = \frac{3}{4}$$

$$\cos x = \pm \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

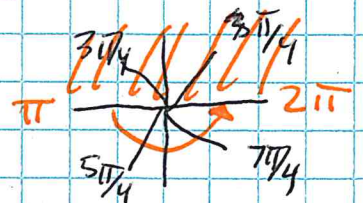


$$c) 1 - 4\sin^2 x = -1; \pi \leq x < 2\pi$$

$$\sin^2 x = \frac{1}{2}$$

$$\sin x = \pm \frac{\sqrt{2}}{2} \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$x = \frac{5\pi}{4}, \frac{7\pi}{4}$$

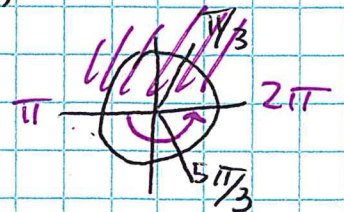


$$d) 3 + 2\cos x = 4; \pi \leq x < 2\pi$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$x = \frac{5\pi}{3}$$





4. a)  $\sin(3x) = -1$

$$\frac{3x}{3} = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2} \Rightarrow x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

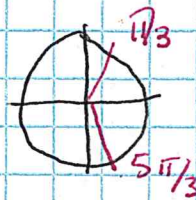


b)  $6 \cos(x - \pi/4) = 3$

$$\cos(x - \pi/4) = 1/2$$

$$x - \pi/4 = (\pi/3, 5\pi/3) + \pi/4$$

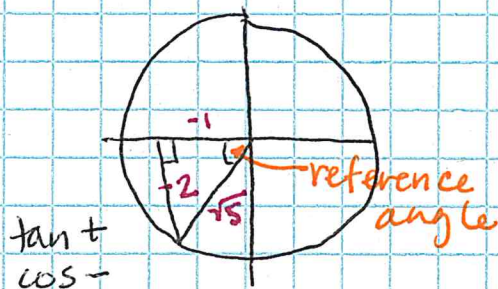
$$x = \frac{7\pi}{12}, \frac{23\pi}{12}$$



$$\frac{\pi}{3} + \pi/4 = \frac{4\pi}{12} + \frac{3\pi}{12} = \frac{7\pi}{12}$$

$$5\pi/3 + \pi/4 = \frac{20\pi}{12} + \frac{3\pi}{12}$$

5.  $\tan x = 2$ ;  $\cos x < 0$



$$\cot x = 1/2$$

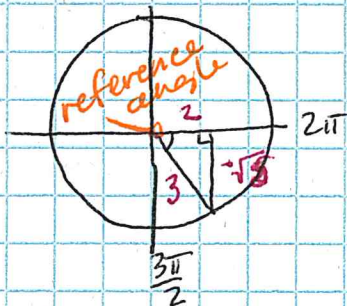
$$\sin x = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$$

$$\csc x = \frac{-\sqrt{5}}{2}$$

$$\cos x = \frac{-1}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$$

$$\sec x = -\sqrt{5}$$

6.  $\sec x = \frac{3}{2}$ ;  $\frac{3\pi}{2} < x < 2\pi$



$$\cos x = \frac{2}{3}$$

$$\tan x = \frac{-\sqrt{13}}{2}$$

$$\sin x = \frac{-\sqrt{13}}{3}$$

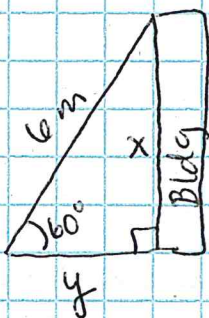
$$\cot x = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$$

$$\csc x = \frac{-3}{\sqrt{13}} = \frac{-3\sqrt{13}}{13}$$

$$9-4=5$$

$$5+4=9$$

7.



$$\sin 60^\circ = \frac{x}{6m}$$

$$\frac{\sqrt{3}}{2} = \frac{x}{6m}$$

$$x = \sqrt{3} \cdot 3m$$

$$\approx 5.196m$$

$$\approx 5.20m$$

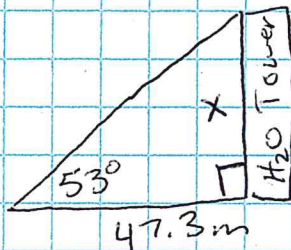
$$\cos 60^\circ = \frac{y}{6m}$$

$$\frac{1}{2} = \frac{y}{6}$$

$$y = 3m$$



8.

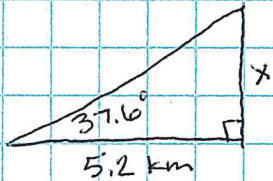


$$\tan 53^\circ = \frac{x}{47.3 \text{ m}}$$

$$x = 47.3 \tan 53^\circ$$

$$x \approx 62.77 \text{ m}$$

9.



$$\tan 37.6^\circ = \frac{x}{5.2 \text{ km}}$$

$$x = (5.2 \text{ km}) (\tan 37.6^\circ)$$

$$x \approx 4.00 \text{ km}$$

$$10. y = 1 + 4 \cot \left( \frac{x}{2} + \frac{\pi}{6} \right)$$

$$a = 4$$

$$b = \frac{1}{2}$$

$$h = \text{left } \frac{\pi}{3}$$

$$k = 1$$

$$\text{period} = 2\pi$$

$$p \cdot b = \pi$$

(for cot/tan)

$$p \cdot \frac{1}{2} = \pi$$

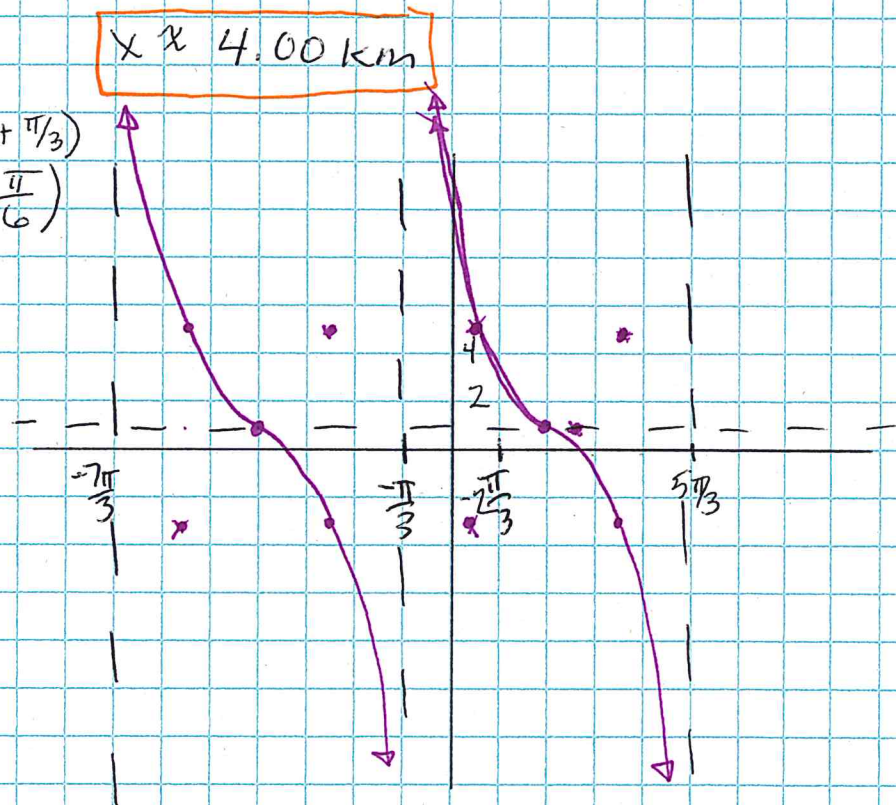
$$p = 2\pi$$

Vert. Asy.  $\cot = \frac{\cos}{\sin}$

$$\sin \left( \frac{x}{2} + \frac{\pi}{6} \right) = 0$$

$$\frac{x}{2} + \frac{\pi}{6} = 0, \pi, 2\pi, -\pi, -2\pi \Rightarrow \frac{x}{2} = -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}, -\frac{7\pi}{6}, -\frac{13\pi}{6}$$

$$\Rightarrow x = -\frac{\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{3}, -\frac{7\pi}{3}, -\frac{13\pi}{3}$$





$$10. f(x) = 3 \sec\left(2x + \frac{3\pi}{2}\right) + 2$$

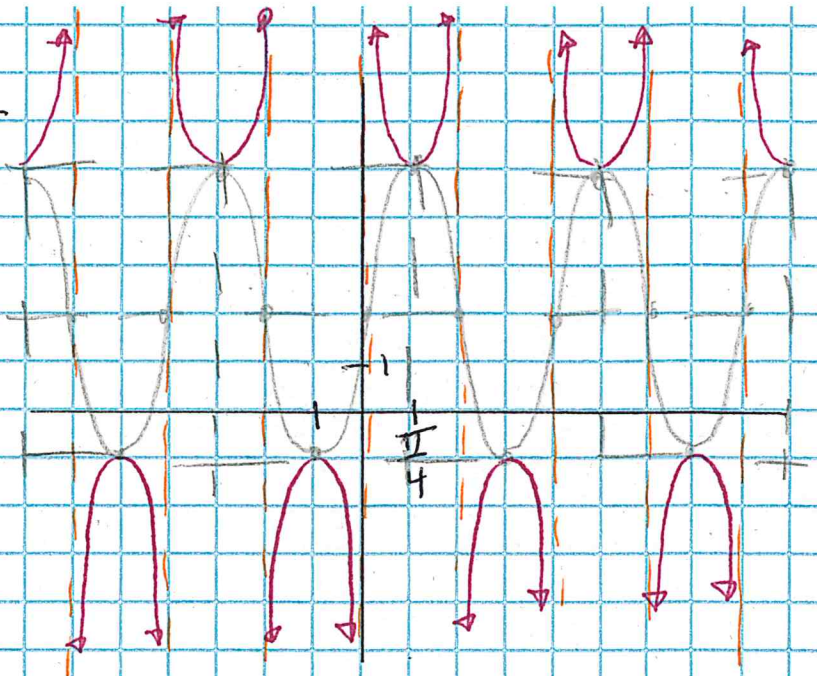
$$a = 3$$

$$b = 2$$

$$h = -\frac{3\pi}{4}$$

$$k = 2$$

$$\text{period} = \pi = \frac{4\pi}{4}$$



$$f(x) = -2 + 3 \csc\left(\frac{x}{4} + \frac{3\pi}{8}\right)$$

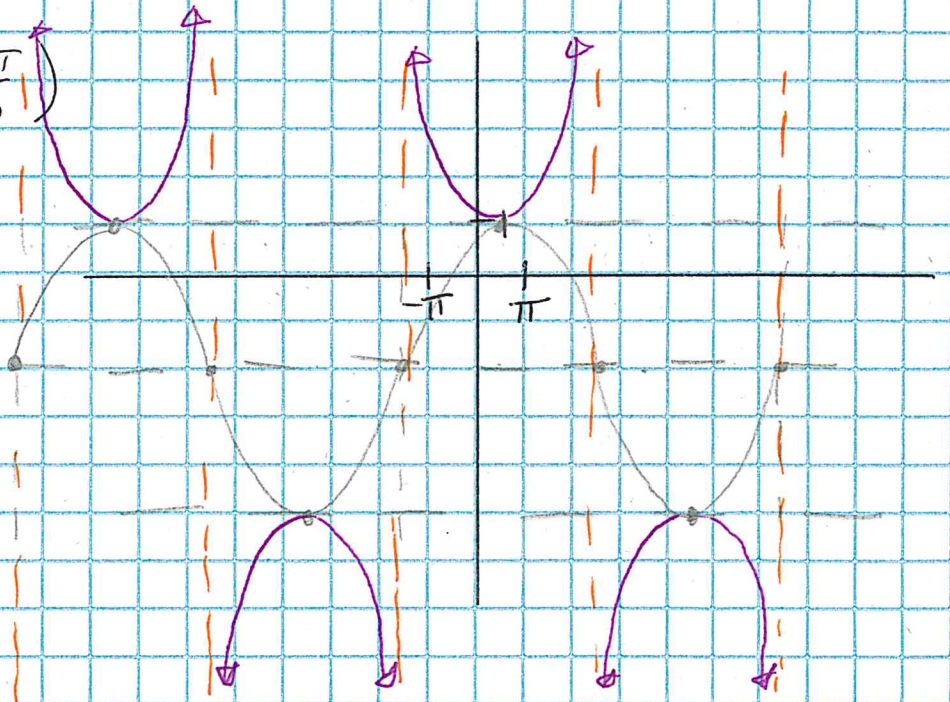
$$a = 3$$

$$b = \frac{1}{4}$$

$$h = -\frac{3\pi}{2}$$

$$k = -2$$

$$\text{period} = 8\pi = \frac{16\pi}{2}$$



$$y = \frac{1}{4} \tan\left(4x + \frac{4\pi}{3}\right) - 1$$

$$a = \frac{1}{4}$$

$$b = 4$$

$$h = -\frac{\pi}{3}$$

$$k = -1$$

$$\text{period} = \frac{\pi}{4} = \frac{6\pi}{24}$$

$$p.b = \pi$$

$$p.4 = \pi$$

$$p = \frac{\pi}{4}$$

$$\cos\left(4x + \frac{4\pi}{3}\right) = 0$$

$$4x + \frac{4\pi}{3} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \quad x = -\frac{5\pi}{12}, \frac{\pi}{12}$$

