

Pre-Calculus Semester 2 Final Review

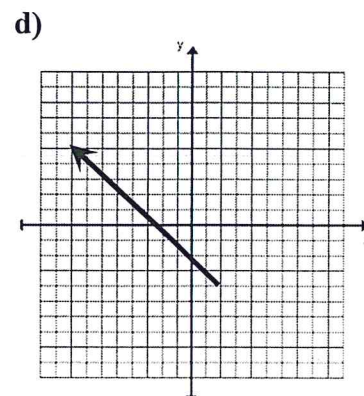
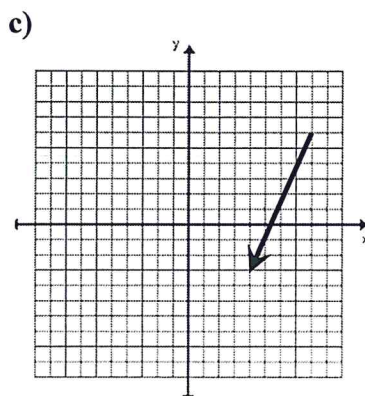
Vectors

- Write vectors in component form.
- Find magnitude and direction from a given vector.
- Vector operations (+, -, ×, ÷)

1. Write the given vector in component form.

a) $\|v\| = 12$ with direction $\theta = 122^\circ$

b) $\|u\| = 2$ with direction $\theta = 307^\circ$



2. Find direction and magnitude of each vector.

a) $v = \langle -2, 7 \rangle$

b) $v = \langle 1, -8 \rangle$

c) $v = \langle -11, -5 \rangle$

d) $v = \langle 7, 3 \rangle$

3. Let $v = -3i + 8j$ and $w = 10i - 4j$.

a) $v + w$

b) $w - v$

c) $3v + 2w$

d) $-4v - w$

Polar Equations

- Convert between polar and rectangular coordinates.
- Graph in polar coordinates.

4. Convert from polar coordinates to rectangular coordinates.

a) $r = 3 \cos \theta$

b) $r = \cos 3\theta + \sin 2\theta$

c) $(-3, 70^\circ)$

d) $(9, 190^\circ)$

5. Graph each equation. Pay attention to the θ -range used.

a) $r = -4 \cos 6\theta$

b) $r = 2 - 8 \sin \theta$

c) $r = \frac{-4}{\cos \theta + \sin \theta}$

d) $r^2 = \frac{\theta}{4}$

Parametric Equations

- Convert parametric equations to rectangular form..
- Graph parametric equations.

6. Graph each equation (pay attention to the t -range used). Then convert each parametric equation to rectangular form.

a) $x(t) = t^2 + 3t$
 $y(t) = t - 1$

b) $x(t) = t^2$
 $y(t) = t^2 - 4t$

Sequences & Series

- Write the explicit and/or recursive equations for a given sequence.
- Given the explicit or recursive equation, write the sequence
- Given Sigma (summation) Notation, write the series.
- Write a given series in sigma notation.
- Calculate finite and infinite sums.

7. Write the explicit and recursive formulas for each sequence.

a) 3, -6, 12, -24, 48, ...

b) 27, 19, 11, 3, ..., -101

c) $\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \dots, \frac{1}{132}$

8. Write the first five terms of each sequence.

a) $a_n = (-1)^n (3n); n \geq 0$

b) $a_1 = 7$
 $a_n = 4 + 2a_{n-1} \quad n \geq 2$

c) $a_1 = 2$
 $a_n = n + a_{n-1} \quad n \geq 2$

9. Write each series in sigma notation.

a) $-1 + \frac{2}{3} - \frac{4}{9} + \frac{8}{37} \dots$

b) $9 + 6 + 3 + 0 \dots - 48$

c) $\frac{1}{2} + 2 + \frac{9}{2} + 8 + \frac{25}{2} + 18 \dots$

10. Evaluate.

d) $\sum_{n=1}^{82} \left(200 - \frac{5}{2}(n-1) \right)$

e) $\sum_{n=1}^{30} \frac{1}{6}(2)^{n-1}$

f) $\sum_{n=1}^{\infty} -5 \left(\frac{3}{2} \right)^{n-1}$

g) $\sum_{n=1}^{\infty} 9 \left(\frac{2}{3} \right)^{n-1}$

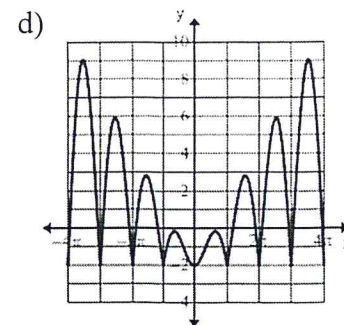
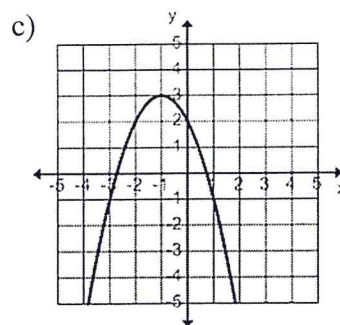
General Topics

- Find domain and range from an equation or graph.
- Describe transformation of parent functions given an equation or graph.

11. Find domain and range of the given function.

a) $y = \frac{\sqrt{x-4}}{x+1}$

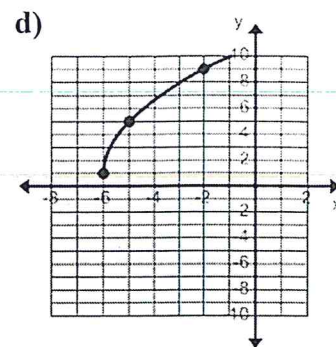
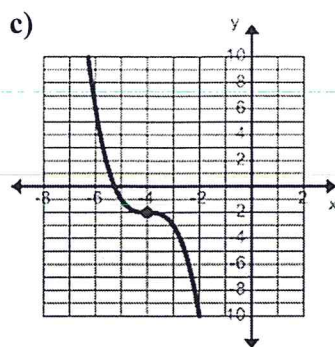
b) $y = -2e^{-x} + 4$



12. Describe the transformations represented by the equation or graph below.

a) $y = 2 \sin\left(3x + \frac{6\pi}{5}\right)$

b) $y = -\frac{1}{5}f(x+3) - 7$



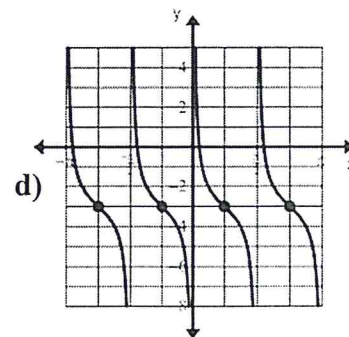
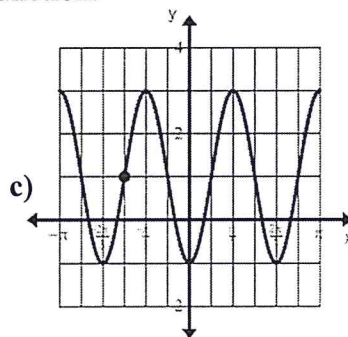
Trigonometric Functions

- State amplitude, period, and phase shift from an equation or graph.
- Write the equation of a sinusoidal function given a transformations or a graph.
- Solve trig equations on a given interval
- Given a trig value, use identities to find another value.

13. State the amplitude, period, and phase shift of each function.

a) $y = 3 \sec(2x - \pi)$

b) $y = -\frac{5}{3} \tan(\pi x + 6\pi)$



14. Write an equation with the given characteristics.

- a)
- Parent : $y = \sin x$
 - amplitude : 2
 - period : $\frac{\pi}{4}$
 - phase shift : right π

- b)
- Parent : $y = \cot x$
 - amplitude : $\frac{3}{4}$
 - period : 2
 - phase shift : left 4

15. Solve for x in the interval $[-\pi, 2\pi]$.

a) $2 \cot x + 1 = -3$

b) $\csc(2x) = 4$

c) $-\tan x = 2 \cos x$

d) $\sec \frac{1}{2}x = 3 \sin \frac{1}{2}x$

16. Given that $\cos x = \frac{5}{8}$ and $\sin x > 0$, find $\tan x$.

Conics

- For parabolas, ellipses, and hyperbolas, identify the center, vertices, foci and sketch the graph.
- Write the equation of a given conic in standard form from given characteristics.
- Write the equation in standard form from general form.

17. Identify the center, vertices, foci and graph the equation.

a) $\frac{(x+3)^2}{49} + \frac{(y-4)^2}{25} = 1$ b) $\frac{(y-6)^2}{9} - \frac{(x-1)^2}{36} = 1$ c) $\frac{(y+1)^2}{4} = (x-2)$ d) $\frac{(x+5)^2}{4} + \frac{(y-2)^2}{4} = 1$

18. Identify the conic section represented. Then write the equation in standard form.

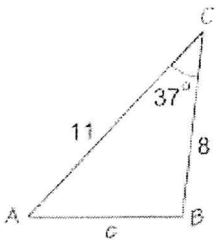
a) $x^2 + y^2 - 4x - 6y + 13 = 0$ b) $2x^2 - 2y^2 - 8x + 12y - 8 = 0$ c) $4y^2 - 24y - 40x - 4 = 0$

More Trigonometry

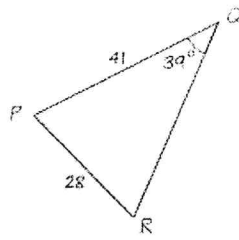
- Use trig ratios and the Laws of Sines and Cosines to find missing side lengths and angle measurements.
- Use trig identities to simplify expressions and calculate values.

19. Find all missing side lengths and angles of each triangle described.

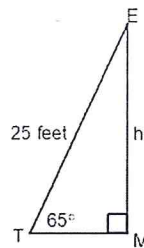
a)



b)



c)



d) Three sides of a triangle measure 20m, 30m, and 40m.

$$m\angle A = 65^\circ$$

e) $a = 18$

$$b = 22$$

20. Find $\cos 2\theta$ and $\cos \frac{\theta}{2}$ if $\cos \theta = -\frac{2}{3}$ and θ is in Quadrant III.

21. Simplify the expression.

a) $\sin x + \cos x \cot x$

b) $\frac{\sec x - \cos x}{\sec x}$

c) $\tan x + \frac{\cos x}{1 + \sin x}$

d) $\frac{\sec x \sin^2 x}{1 + \sec x}$