How to Graph a Sinusoidal Function:

General Forms	$y = \boldsymbol{a}\sin\boldsymbol{b}(x - \boldsymbol{h}) + \boldsymbol{k}$	$y = \boldsymbol{a}\cos\boldsymbol{b}(x - \boldsymbol{h}) + \boldsymbol{k}$

The easiest way to draw accurate sinusoidal graphs is to create "boxes" of dotted lines across the domain. Each box represents one complete cycle.

1) Identify the following information: ***a*** (amplitude) =
b (frequency) =
period (use $p \cdot b = 2\pi$) =
h ("phase shift" – shift left /right) =
k ("axis" – shift up/down) =

2) Graph the axis based on ***k***. Everything is based off the axis. If there is no vertical translation, the axis remains $y = 0$.

3) Mark the amplitude with a horizontal dotted line above and below the axis. The entire graph fits between these two dotted lines.

4) Determine your scale based on the phase shift and the period. You should choose a scale that makes graphing both of these values easy.

Examples

Phase shift & period	*Scale*	*Reason*
left $\frac{\pi}{3}$ with a period of $\frac{3\pi}{2}$	$\frac{\pi}{6}$	6 is the common denominator making both the phase shift and period fall on a "nice" axis mark.
left $\frac{\pi}{3}$ with a period of $\frac{7\pi}{4}$	$\frac{\pi}{12}$	12 is the common denominator making both the phase shift and period fall on a "nice" axis mark.

5) Mark the amplitude with a dotted vertical line. This marks the beginning of the first "box".

6) From the phase shift, count the period along the x-axis. Mark the end of one period with another dotted vertical line. This marks the end of the first box. More boxes can be marked in this way in both directions.

7) Using the appropriate point pattern for either sine or cosine, plot the key points within each box.

8) Connect the dots with a smooth curve.

Here's an example. Graph $y = 3\cos 2x - 4$

Step 1 – Identify Info ***a* = 3**

***b* = 2**

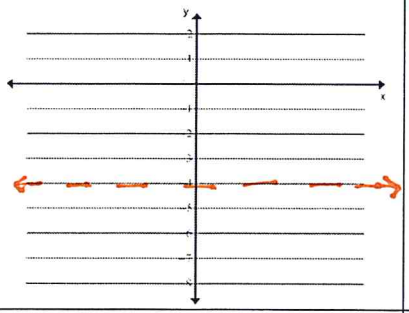
period = π To find period: $pb = 2\pi \rightarrow 2p = 2\pi \rightarrow p = \pi$

h = 0 (or none)

k = down 4 (axis is $y = -4$)

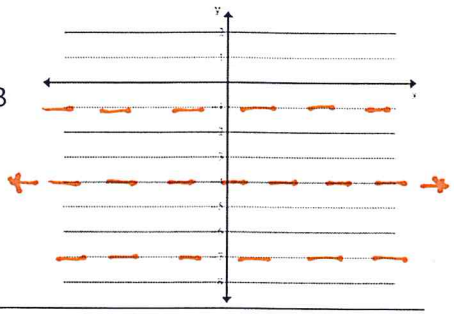
Step 2 – Axis

$y = -4$



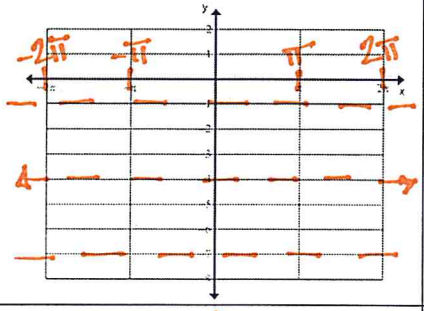
Step 3 – Amplitude

$a = 3$



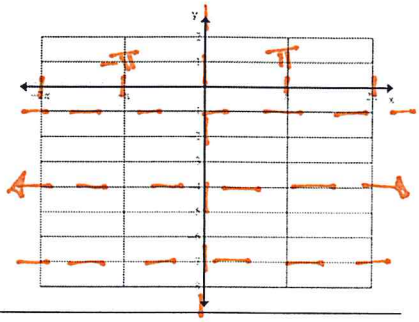
Step 4 – Choose Scale

No phase shift with a period of π means π is good scale.



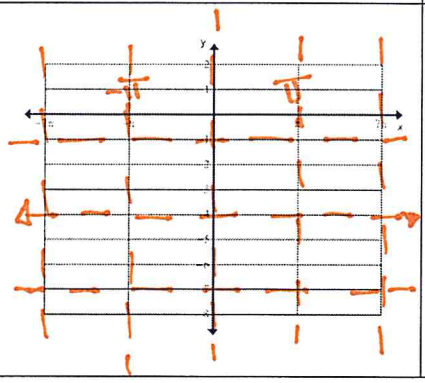
Step 5 – Phase Shift

No phase shift means the cycle begins at $x = 0$.



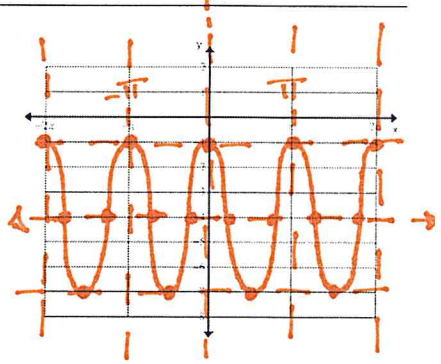
Step 6 – Period

period = π



Step 7 – Plot Key Points

Step 8 – Draw the Curve



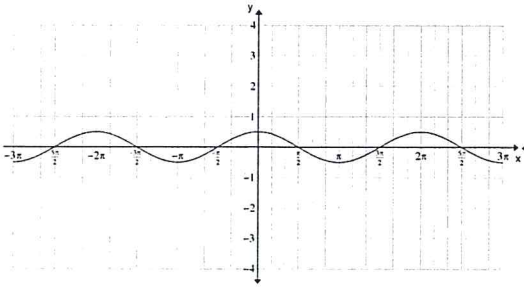
Homework:

Graph the following each function.

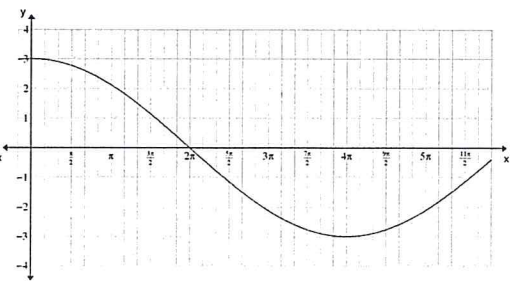
1. $y = \cos(3x)$
2. $y = \sin(4x)$
3. $y = \cos\left(\frac{x}{2}\right)$
4. $y = \sin\left(\frac{x}{3}\right)$
5. $y = \frac{1}{2}\cos(x)$
6. $y = 4\sin(x)$
7. $y = 3\sin x - 1$
8. $y = 2\cos(x) + 4$
9. $y = 2\cos\left(\frac{x}{2}\right) - 3$
10. $y = -\sin(2x) + 1$
11. $y = -4\cos\left(\frac{2}{3}x\right) + 2$
12. $y = \frac{7}{2}\sin\left(\frac{4}{3}x\right) - 2$

Find the equation of the given graph using the sinusoidal parent function given.

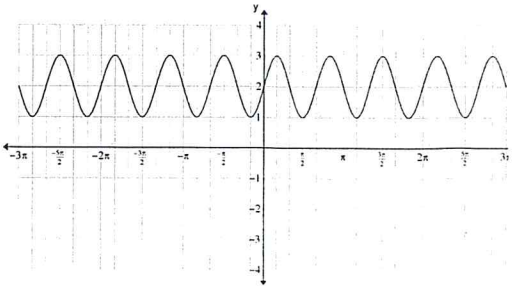
13. $y = \cos x$



14. $y = \cos x$

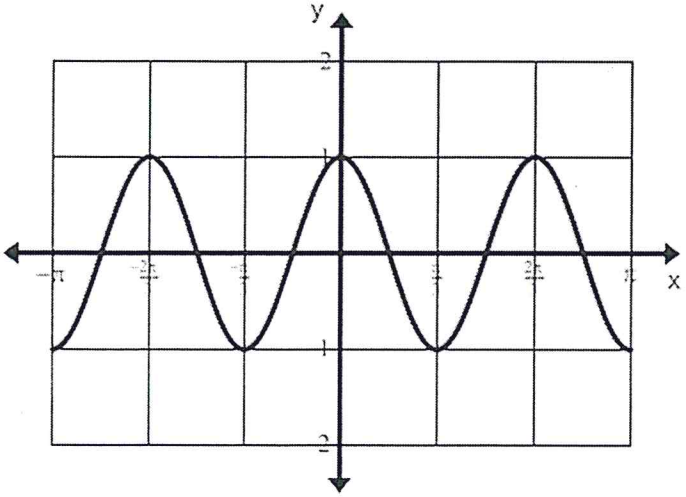


15. $y = \sin x$



Answers:

1.



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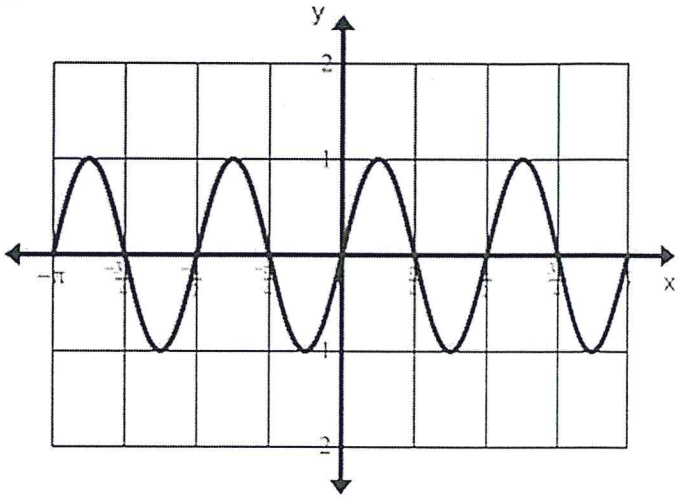

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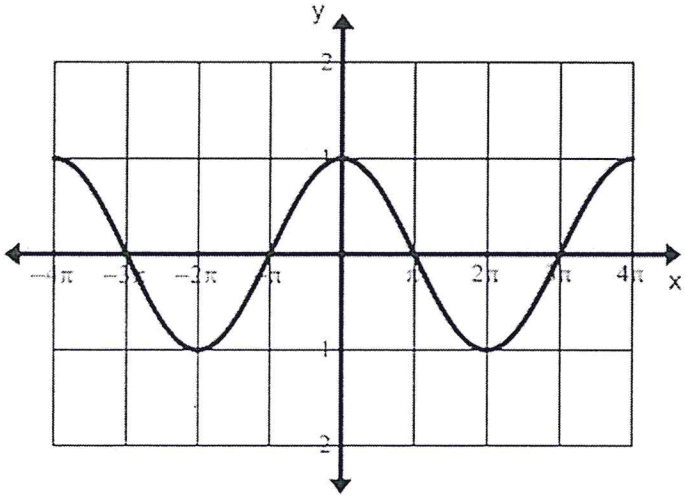

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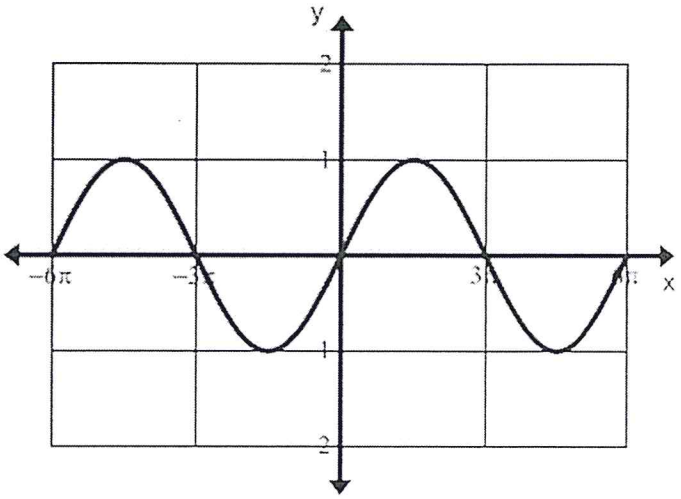

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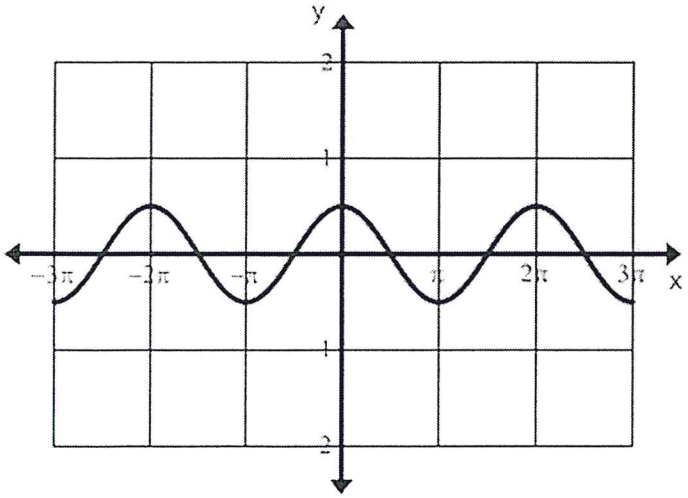

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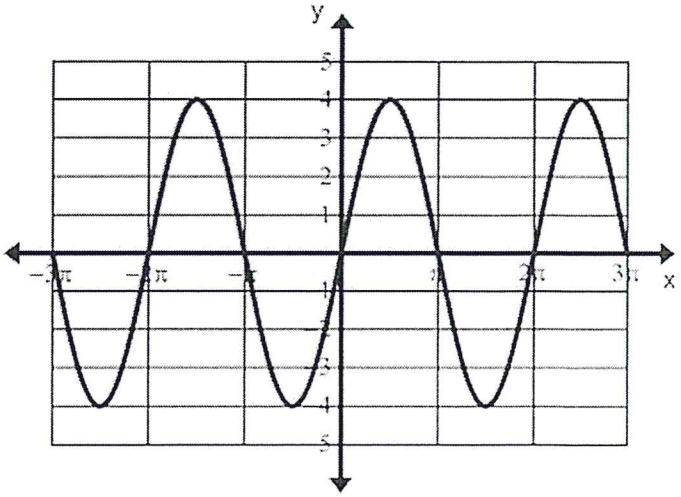

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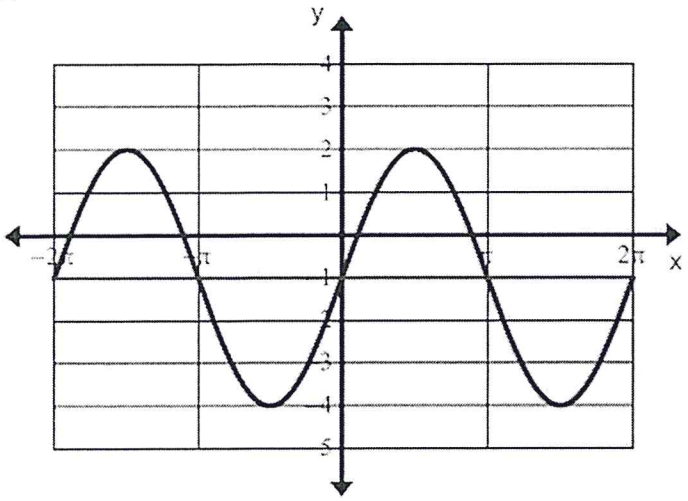

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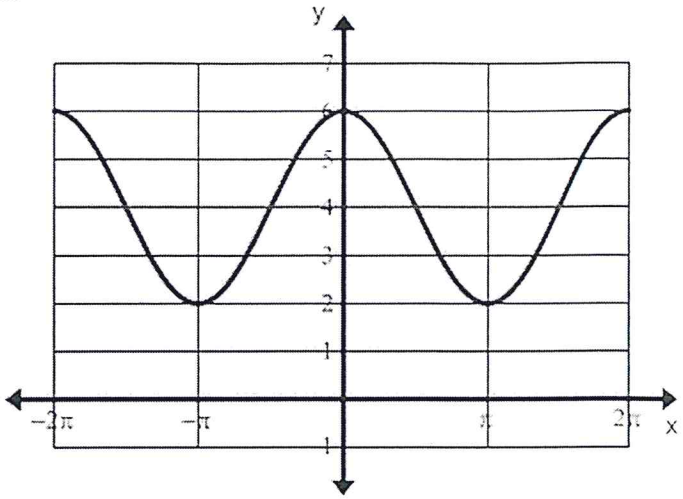

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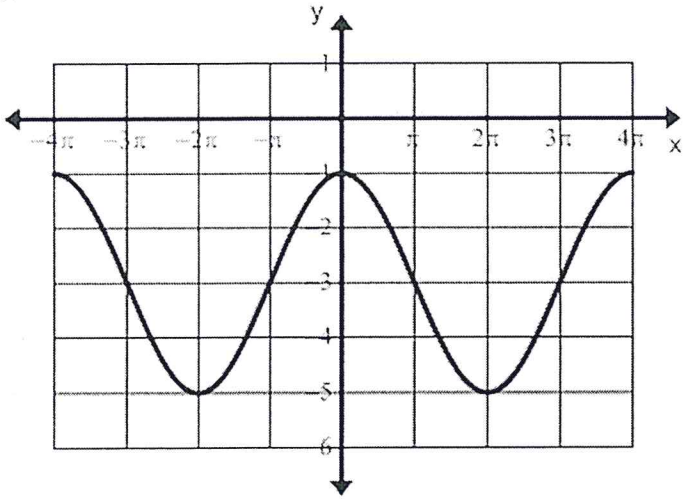

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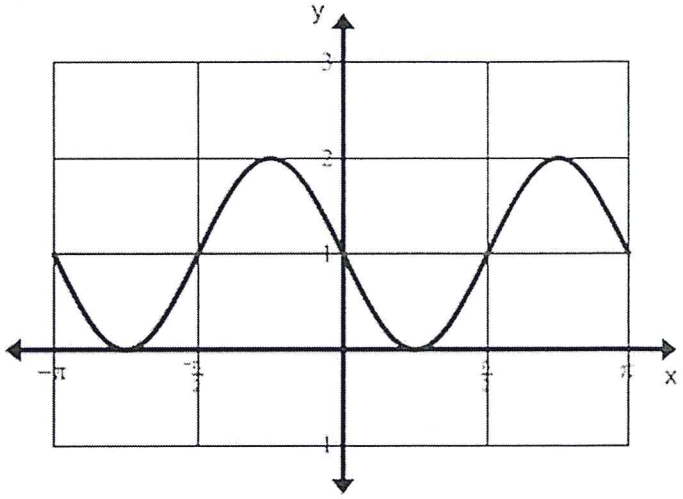

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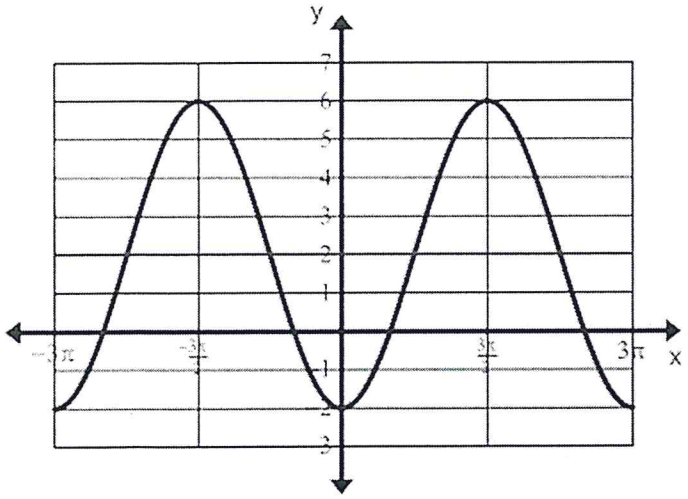

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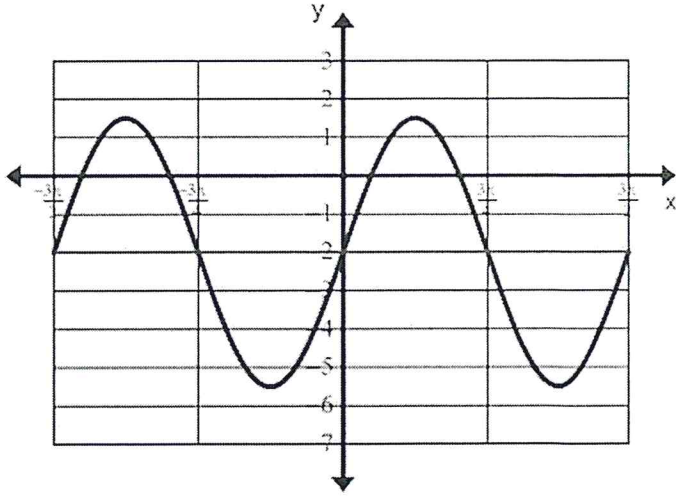

13. $y = \frac{1}{2}\cos x$

14. $y = 3\cos\left(\frac{x}{4}\right)$

15. $y = \sin(3x) + 2$