Chapter 2 Polynomial, Power, and Rational Functions

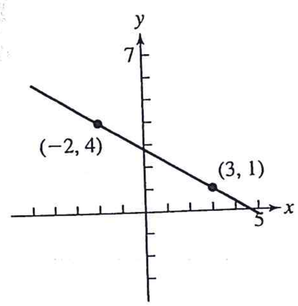
Section 2.1 Linear and Quadratic Functions and Modeling

Exploration 1

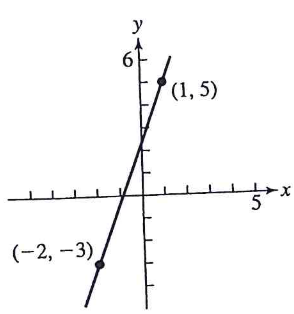
1. −\$2000 per year
2. The equation will have the form $v(t) = mt + b$. The value of the building after 0 year is $v(0) = m(0) + b = b = 50{,}000$.
   The slope m is the rate of change, which is −2000 (dollars per year). So an equation for the value of the building (in dollars) as a function of the time (in years) is $v(t) = -2000t + 50{,}000$.
3. $v(0) = 50{,}000$ and $v(16) = -2000(16) + 50{,}000 = 18{,}000$
4. The equation $v(t) = 39{,}000$ becomes
   $$-2000t + 50{,}000 = 39{,}000$$
   $$-2000t = -11{,}000$$
   $$t = 5.5$$

Quick Review 2.1

1. $y = 8x + 3.6$
2. $y = -1.8x - 2$
3. $y - 4 = -\frac{3}{5}(x + 2)$, or $y = -0.6x + 2.8$



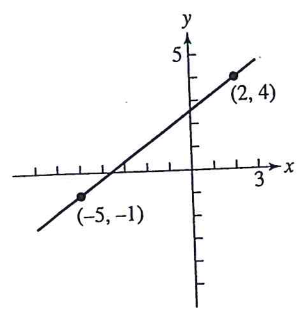
4. $y - 5 = \frac{8}{3}(x - 1)$, or $y = \frac{8}{3}x + \frac{7}{3}$



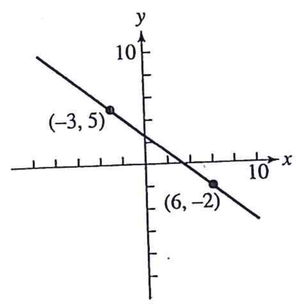
5. $(x + 3)^2 = (x + 3)(x + 3) = x^2 + 3x + 3x + 9$
   $= x^2 + 6x + 9$
6. $(x - 4)^2 = (x - 4)(x - 4) = x^2 - 4x - 4x + 16$
   $= x^2 - 8x + 16$
7. $3(x - 6)^2 = 3(x - 6)(x - 6) = (3x - 18)(x - 6)$
   $= 3x^2 - 18x - 18x + 108 = 3x^2 - 36x + 108$
8. $-3(x + 7)^2 = -3(x + 7)(x + 7)$
   $= (-3x - 21)(x + 7) = -3x^2 - 21x - 21x - 147$
   $= -3x^2 - 42x - 147$
9. $2x^2 - 4x + 2 = 2(x^2 - 2x + 1) = 2(x - 1)(x - 1)$
   $= 2(x - 1)^2$
10. $3x^2 + 12x + 12 = 3(x^2 + 4x + 4) = 3(x + 2)(x + 2)$
    $= 3(x + 2)^2$

Section 2.1 Exercises

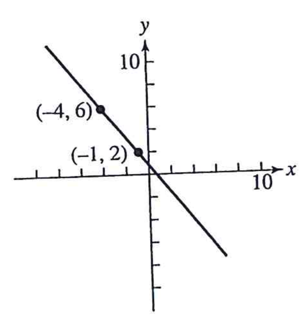
1. Not a polynomial function because of the exponent −5.
2. Polynomial of degree 1 with leading coefficient 2.
3. Polynomial of degree 5 with leading coefficient 2.
4. Polynomial of degree 0 with leading coefficient 13.
5. Not a polynomial function because of cube root.
6. Polynomial of degree 2 with leading coefficient −5.
7. $m = \frac{5}{7}$ so $y - 4 = \frac{5}{7}(x - 2) \Rightarrow f(x) = \frac{5}{7}x + \frac{18}{7}$



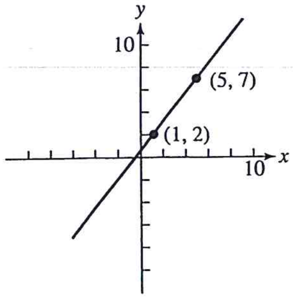
8. $m = -\frac{7}{9}$ so $y - 5 = -\frac{7}{9}(x + 3) \Rightarrow f(x) = -\frac{7}{9}x + \frac{8}{3}$



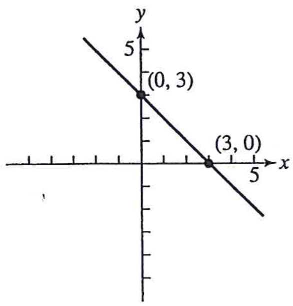
9. $m = -\frac{4}{3}$ so $y - 6 = -\frac{4}{3}(x + 4) \Rightarrow f(x) = -\frac{4}{3}x + \frac{2}{3}$



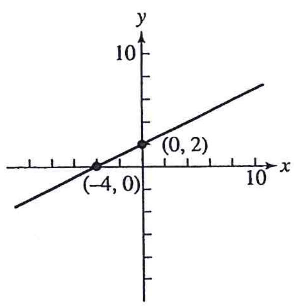
10. $m = \frac{5}{4}$ so $y - 2 = \frac{5}{4}(x - 1) \Rightarrow f(x) = \frac{5}{4}x + \frac{3}{4}$



11. $m = -1$ so $y - 3 = -1(x - 0) \Rightarrow f(x) = -x + 3$



12. $m = \frac{1}{2}$ so $y - 2 = \frac{1}{2}(x - 0) \Rightarrow f(x) = \frac{1}{2}x + 2$



13. (a)—the vertex is at $(-1, -3)$, in quadrant III, eliminating all but (a) and (d). Since $f(0) = -1$, it must be (a).

14. (d)—the vertex is at $(-2, -7)$, in quadrant III, eliminating all but (a) and (d). Since $f(0) = 5$, it must be (d).

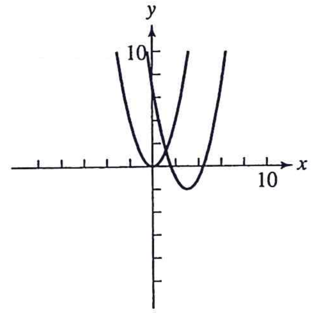
15. (b)—the vertex is in quadrant I, at $(1, 4)$, meaning it must be either (b) or (f). Since $f(0) = 1$, it cannot be (f): if the vertex in (f) is $(1, 4)$, then the intersection with the y-axis would be about $(0, 3)$. It must be (b).

16. (f)—the vertex is in quadrant I, at $(1, 12)$, meaning it must be either (b) or (f). Since $f(0) = 10$, it cannot be (b): if the vertex in (b) is $(1, 12)$, then the intersection with the y-axis occurs considerably lower than $(0, 10)$. It must be (f).

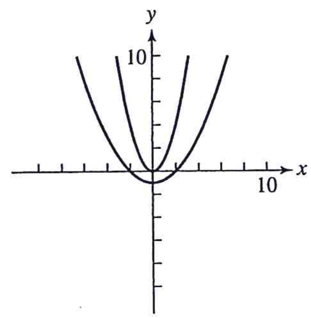
17. (e)—the vertex is at $(1, -3)$ in Quadrant IV. So it must be (e).

18. (c)—the vertex is at $(-1, 12)$ in Quadrant II and the parabola opens down, so it must be (c).

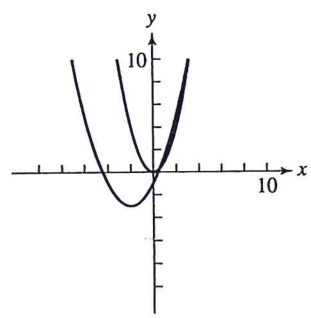
19. Translate the graph of $f(x) = x^2$ 3 units right to obtair the graph of $h(x) = (x - 3)^2$, and translate this graph 2 units down to obtain the graph of $g(x) = (x - 3)^2 -$



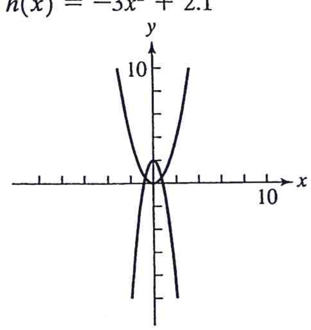
20. Vertically shrink the graph of $f(x) = x^2$ by a factor of $\frac{1}{4}$ to obtain the graph of $g(x) = \frac{1}{4}x^2$, and translate this graph 1 unit down to obtain the graph of $h(x) = \frac{1}{4}x^2 -$



21. Translate the graph of $f(x) = x^2$ 2 units left to obtain tl graph of $h(x) = (x + 2)^2$, vertically shrink this graph t a factor of $\frac{1}{2}$ to obtain the graph of $k(x) = \frac{1}{2}(x + 2)^2$ and translate this graph 3 units down to obtain the gra of $g(x) = \frac{1}{2}(x + 2)^2 - 3$.



22. Vertically stretch the graph of $f(x) = x^2$ by a factor o to obtain the graph of $g(x) = 3x^2$, reflect this graph across the x-axis to obtain the graph of $k(x) = -3x^2$, translate this graph up 2 units to obtain the graph of $h(x) = -3x^2 + 2.1$



For #23–32, with an equation of the form $f(x) = a(x - h)^2 + k$, the vertex is (h, k) and the axis is $x = h$.

23. Vertex: $(1, 5)$; axis: $x = 1$

24. Vertex: $(-2, -1)$; axis: $x = -2$

25. Vertex: $(1, -7)$; axis: $x = 1$

26. Vertex: $(\sqrt{3}, 4)$; axis: $x = \sqrt{3}$

27. $f(x) = 3\left(x^2 + \frac{5}{3}x\right) - 4$

$= 3\left(x^2 + 2\cdot\frac{5}{6}x + \frac{25}{36}\right) - 4 - \frac{25}{12} = 3\left(x + \frac{5}{6}\right)^2 - \frac{73}{12}$

Vertex: $\left(-\frac{5}{6}, -\frac{73}{12}\right)$; axis: $x = -\frac{5}{6}$

28. $f(x) = -2\left(x^2 - \frac{7}{2}x\right) - 3$

$= -2\left(x^2 - 2\cdot\frac{7}{4}x + \frac{49}{16}\right) - 3 + \frac{49}{8}$

$= -2\left(x - \frac{7}{4}\right)^2 + \frac{25}{8}$

Vertex: $\left(\frac{7}{4}, \frac{25}{8}\right)$; axis: $x = \frac{7}{4}$

29. $f(x) = -(x^2 - 8x) + 3$

$= -(x^2 - 2\cdot 4x + 16) + 3 + 16 = -(x - 4)^2 + 19$

Vertex: $(4, 19)$; axis: $x = 4$

30. $f(x) = 4\left(x^2 - \frac{1}{2}x\right) + 6$

$= 4\left(x^2 - 2\cdot\frac{1}{4}x + \frac{1}{16}\right) + 6 - \frac{1}{4} = 4\left(x - \frac{1}{4}\right)^2 + \frac{23}{4}$

Vertex: $(0.25, 5.75)$; axis: $x = 0.25$

31. $g(x) = 5\left(x^2 - \frac{6}{5}x\right) + 4$

$= 5\left(x^2 - 2\cdot\frac{3}{5}x + \frac{9}{25}\right) + 4 - \frac{9}{5} = 5\left(x - \frac{3}{5}\right)^2 + \frac{11}{5}$

Vertex: $(0.6, 2.2)$; axis: $x = 0.6$

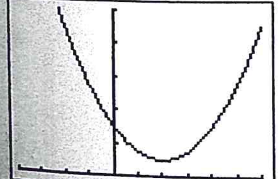
32. $h(x) = -2\left(x^2 + \frac{7}{2}x\right) - 4$

$= -2\left(x^2 + 2\cdot\frac{7}{4}x + \frac{49}{16}\right) - 4 + \frac{49}{8}$

$= -2\left(x + \frac{7}{4}\right)^2 + \frac{17}{8}$

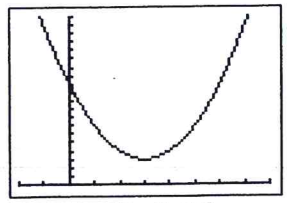
Vertex: $(-1.75, 2.125)$; axis: $x = -1.75$

33. $f(x) = (x^2 - 4x + 4) + 6 - 4 = (x - 2)^2 + 2$.
Vertex: $(2, 2)$; axis: $x = 2$; opens upward; does not intersect x-axis.



$[-4, 6]$ by $[0, 20]$

34. $g(x) = (x^2 - 6x + 9) + 12 - 9 = (x - 3)^2 + 3$.
Vertex: $(3, 3)$; axis: $x = 3$; opens upward; does not intersect x-axis.

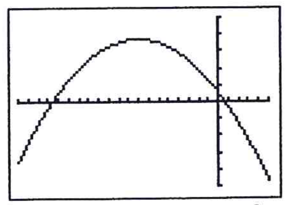


$[-2, 8]$ by $[0, 20]$

35. $f(x) = -(x^2 + 16x) + 10$

$= -(x^2 + 16x + 64) + 10 + 64 = -(x + 8)^2 + 74$.

Vertex: $(-8, 74)$; axis: $x = -8$; opens downward; intersects x-axis at about -16.602 and $0.602 (-8 \pm \sqrt{74})$.

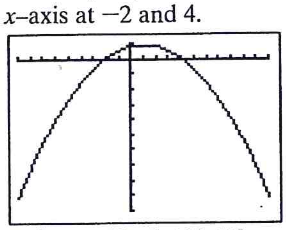


$[-20, 5]$ by $[-100, 100]$

36. $h(x) = -(x^2 - 2x) + 8 = -(x^2 - 2x + 1) + 8 + 1$

$= -(x - 1)^2 + 9$

Vertex: $(1, 9)$; axis: $x = 1$; opens downward; intersects x-axis at -2 and 4.

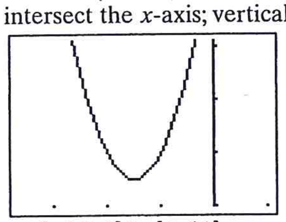


$[-9, 11]$ by $[-100, 10]$

37. $f(x) = 2(x^2 + 3x) + 7$

$= 2\left(x^2 + 3x + \frac{9}{4}\right) + 7 - \frac{9}{2} = 2\left(x + \frac{3}{2}\right)^2 + \frac{5}{2}$

Vertex: $\left(-\frac{3}{2}, \frac{5}{2}\right)$; $x = -\frac{3}{2}$; opens upward; does not intersect the x-axis; vertically stretched by 2.



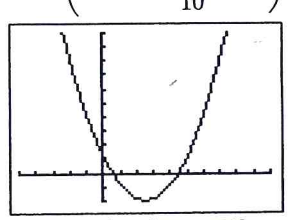
$[-3.7, 1]$ by $[2, 5.1]$

38. $g(x) = 5(x^2 - 5x) + 12$

$= 5\left(x^2 - 5x + \frac{25}{4}\right) + 12 - \frac{125}{4}$

$= 5\left(x - \frac{5}{2}\right)^2 - \frac{77}{4}$

Vertex: $(2.5, -19.25)$; axis: $x = 2.5$; opens upward; intersects x-axis at about 0.538 and $4.462 \left(\text{or } 2.5 \pm \frac{1}{10}\sqrt{385}\right)$; vertically stretched by 5.



$[-5, 10]$ by $[-20, 100]$

For #39–44, use the form $y = a(x - h)^2 + k$, taking the vertex (h, k) from the graph or other given information.

39. $h = -1$ and $k = -3$, so $y = a(x + 1)^2 - 3$. Now substitute $x = 1$, $y = 5$ to obtain $5 = 4a - 3$, so $a = 2$: $y = 2(x + 1)^2 - 3$.

40. $h = 2$ and $k = -7$, so $y = a(x - 2)^2 - 7$. Now substitute $x = 0$, $y = 5$ to obtain $5 = 4a - 7$, so $a = 3$: $y = 3(x - 2)^2 - 7$.

41. $h = 1$ and $k = 11$, so $y = a(x - 1)^2 + 11$. Now substitute $x = 4$, $y = -7$ to obtain $-7 = 9a + 11$, so $a = -2$: $y = -2(x - 1)^2 + 11$.

42. $h = -1$ and $k = 5$, so $y = a(x + 1)^2 + 5$. Now substitute $x = 2$, $y = -13$ to obtain $-13 = 9a + 5$, so $a = -2$: $y = -2(x + 1)^2 + 5$.

43. $h = 1$ and $k = 3$, so $y = a(x - 1)^2 + 3$. Now substitute $x = 0$, $y = 5$ to obtain $5 = a + 3$, so $a = 2$: $y = 2(x - 1)^2 + 3$.

44. $h = -2$ and $k = -5$, so $y = a(x + 2)^2 - 5$. Now substitute $x = -4$, $y = -27$ to obtain $-27 = 4a - 5$, so $a = -\frac{11}{2}$: $y = -\frac{11}{2}(x + 2)^2 - 5$.

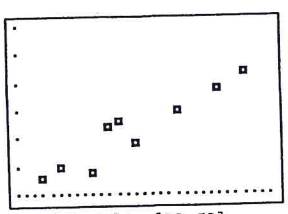
45. Strong positive

46. Strong negative

47. Weak positive

48. No correlation

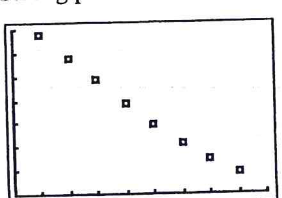
49. (a)



[15, 45] by [20, 50]

(b) Strong positive

50. (a)



[0, 90] by [0, 70]

(b) Strong negative

51. $m = -\frac{2350}{5} = -470$ and $b = 2350$, so $v(t) = -470t + 2350$. At $t = 3$, $v(3) = (-470)(3) + 2350 = \$940$.

52. Let x be the number of dolls produced each week and y be the average weekly costs. Then $m = 4.70$, and $b = 350$, so $y = 4.70x + 350$, or $500 = 4.70x + 350$: $x = 32$, 32 dolls are produced each week.

53. (a) $y \approx 16.395x + 196.888$. The slope, $m \approx 16.395$, represents an increase in y (dollars) for an increase by 1 in the value of x (years). In other words, the slope represents the typical annual increase in weekly earnings.

(b) Setting $x = 35$ in the regression equation leads to $y \approx \$771$.

54. If the length is x, then the width is $50 - x$, so $A(x) = x(50 - x)$; maximum of 625 ft^2 when $x = 25$ (the dimensions are 25 ft $\times$ 25 ft).

55. (a) [0, 100] by [0, 1000] is one possibility.

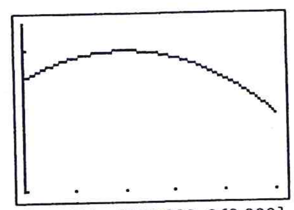
(b) When $x \approx 107.335$ or $x \approx 372.665$ — either 107, 335 units or 372, 665 units.

56. The area of the picture and the frame, if the width of the picture is x ft, is $A(x) = (x + 2)(x + 5)$ ft^2. This equals 208 when $x = 11$, so the painting is 11 ft $\times$ 14 ft.

57. If the strip is x feet wide, the area of the strip is $A(x) = (25 + 2x)(40 + 2x) - 1000$ ft^2. This equals 504 ft^2 when $x = 3.5$ ft.

58. (a) $R(x) = (800 + 20x)(300 - 5x)$.

(b) [0, 25] by [200,000, 260,000] is one possibility (shown).

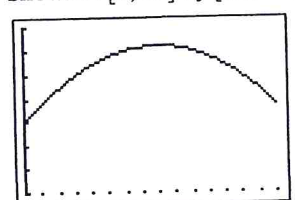


[0, 25] by [200,000, 260,000]

(c) The maximum income — \$250,000 — is achieved when $x = 10$, corresponding to rent of \$250 per month.

59. (a) $R(x) = (26{,}000 - 1000x)(0.50 + 0.05x)$.

(b) Many choices of Xmax and Ymin are reasonable. Shown is [0, 15] by [10,000, 17,000].



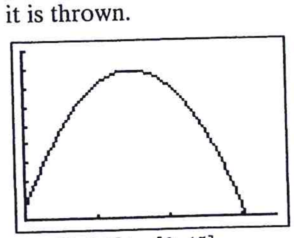
[0, 15] by [10,000, 17,000]

(c) The maximum revenue — \$16,200 — is achieved when $x = 8$; that is, charging 90 cents per can.

60. Total sales would be $S(x) = (30 + x)(50 - x)$ thousand dollars, when x additional salespeople are hired. The maximum occurs when $x = 10$ (halfway between the two zeros, at -30 and 50).

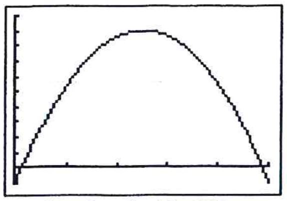
61. (a) $h = -16t^2 + 48t + 3.5$.

(b) The graph is shown in the window [0, 3.5] by [0, 45]. The maximum height is 39.5 ft, 1.5 sec after it is thrown.



[0, 3.5] by [0, 45]

62. **(a)** $h = -16t^2 + 80t - 10$. The graph is shown in the window $[0, 5]$ by $[-10, 100]$.



$[0, 5]$ by $[-10, 100]$

(b) The maximum height is 90 ft, 2.5 sec after it is shot.

63. The exact answer is $32\sqrt{3}$, or about 55.426 ft/sec. In addition to the guess-and-check strategy suggested, this can be found algebraically by noting that the vertex of the parabola $y = ax^2 + bx + c$ has y coordinate $c - \frac{b^2}{4a} = \frac{b^2}{64}$ (note $a = -16$ and $c = 0$), and setting this equal to 48.

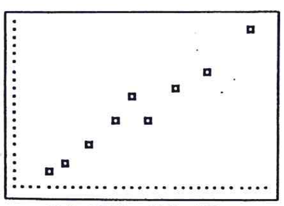
64. $y \approx 0.369x^2 + 1.487x + 113.602$ where $x =$ the number of years after 1980. Solving the equation $0.369x^2 + 1.487x + 113.602 = 350$ graphically, $y = 350$ when $x \approx 23.4$. Patent applications will reach 350,000 in the year 2003.

65. **(a)** $m = \frac{6 \text{ ft}}{100 \text{ ft}} = 0.06$

(b) $r \approx 4167$ ft, or about 0.79 mile.

(c) 2217.6 ft

66. **(a)**

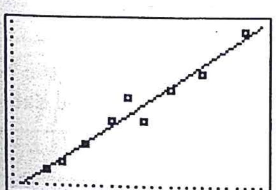


$[15, 45]$ by $[20, 40]$

(b) $y \approx 0.68x + 9.01$

(c) On average, the children gain 0.68 pound per month.

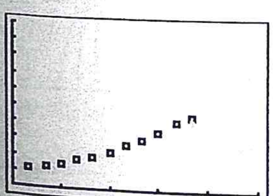
(d)



$[15, 45]$ by $[20, 40]$

(e) ≈ 29.41 lbs

67. **(a)**

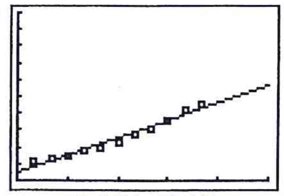


$[0, 15]$ by $[0, 10]$

(b) $y \approx 0.345x + 0.469$, where $x =$ number of years after 1990 in which the season started.

(c) Players' average salaries are increasing by about \$0.345 million $=$ \$345,000 annually.

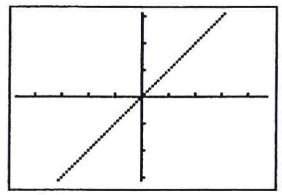
(d)



$[0, 15]$ by $[0, 10]$

(e) When $x = 15$, y equals approximately \$5.6 million $=$ \$5,600,000 annually.

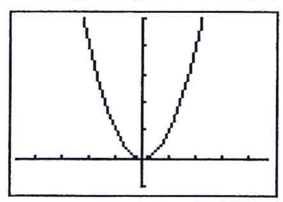
68. The Identity Function $f(x) = x$



$[-4.7, 4.7]$ by $[-3.1, 3.1]$

Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Continuity: The function is continuous on its domain.
Increasing–decreasing behavior: increasing for all x
Symmetry: Symmetric about the origin
Boundedness: Not bounded
Local extrema: None
Horizontal asymptotes: None
Vertical asymptotes: None.
End behavior: $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = \infty$

69. The Squaring Function $f(x) = x^2$



$[-4.7, 4.7]$ by $[-1, 5]$

Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Continuity: The function is continuous on its domain.
Increasing–decreasing behavior: increasing on $[0, \infty)$, decreasing on $(-\infty, 0]$.
Symmetry: Symmetric about the y-axis
Boundedness: Bounded below, but not above
Local extrema: Local minimum of 0 at $x = 0$
Horizontal asymptotes: None
Vertical asymptotes: None
End behavior: $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = \infty$

70. False. For $f(x) = 3x^2 + 2x - 3$, the initial value is $f(0) = -3$.