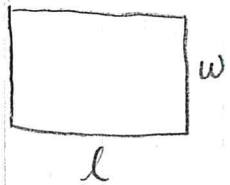
Pre-Calculus Quadratic Modeling Notes/Examples

Maximization/ Minimization

Example 1

A gardener has 140 ft of fencing in a rectangular vegetable garden. Find the dimensions of the largest area she can fence.



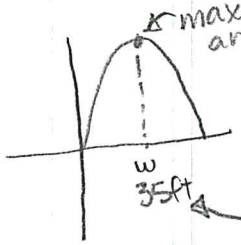
$A = lw$ ← max dimensions?

$2l + 2w = 140$

$2l = 140 - 2w$

$l = 70 - w$

$A = (70-w)w = -w^2 + 70w$



To find vertex, find axis 1st:

$\text{axis} = \frac{-b}{2a} \Rightarrow \frac{-70}{2(-1)} = 35$

$w = 35$

$l = 70 - w = 70 - 35 = 35 \text{ ft}$

dimensions to max area are 35 ft × 35 ft

Example 2

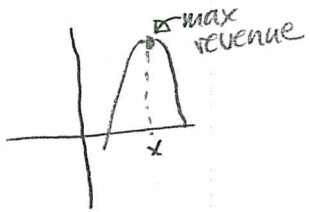
A hockey team plays in an arena with a seating capacity of 15,000 spectators. With the tickets prices set at $14, average attendance at recent games has been 9500. A market survey indicates that for each dollar the ticker price is lowered, the average attendance increases by 1000. Find the ticket price that maximizes revenue from ticket sales.

Ticket price? Revenue ← max

Revenue = (ticket price)(tickets sold)

$= (14 \quad)(9500 \quad)$ ← start point

$= (14 - 1x)(9500 + 1000x)$ ← changes



find axis using roots:

roots are 14, -9.5

$\text{axis} = \frac{14 - 9.5}{2} = \frac{4.5}{2} = 2.25 = x$

Ticket price $= 14 - x = 14 - 2.25 = 11.75$

A price of $11.75 will max revenue.

Objects in Free Fall

The height of an object rising or falling under the influence of gravity is modeled by the function $y = ax^2 + v_0x + s_0$ where x represents time in seconds, y represents the object's height from the ground in meters or feet, a is half the downward acceleration due to gravity (on Earth, a is $-4.9\,\text{m/s}^2$ or $-16\,\text{ft/s}^2$), v_0 is the initial upward velocity of the object and s_0 is the initial height of the object.

Example 1

A soccer ball on the ground was kicked into the air with an initial velocity of 16 meters per second. $a = -4.9$ $V_0 = 16$ m $S_0 = 0$ (on ground)

a. Write the equation which relates distance above where the ball was kicked with respect to time. $y = -4.9x^2 + 16x + 0$

b. After what time(s) was the ball at a height of 12 meters? $y = 12$, find x...

c. How high was the ball after 3 seconds? $x = 3$, find y...

d. How many seconds did it take the ball to hit the ground? find roots.

b. $12 = -4.9x^2 + 16x$

$0 = -4.9x^2 + 16x - 12$

$x = 1.167$ sec & 2.098 sec

(using graph. calc.)

c. $y = -4.9(3)^2 + 16(3)$

$= 3.9$ m

d. $y = -4.9x^2 + 16x$

$= x(-4.9x + 16)$ $x = 3.265$ sec.

$x = 0, 3.265$ sec

start, hits ground

Example 2

An object is launched directly upward at 64 feet per second from a platform 80 feet high. $a = -16$ $V_0 = 64$ $S_0 = 80$

a. What will be the object's maximum height? When will it attain this height? vert

b. How long will it take for the object to reach the ground? roots

c. When will the object be 108 feet above the ground? $y = 108$, find x

d. How long will it take for the object to be 128 feet above the ground? $y = 128$, fi

e. How high will the object be after 4 seconds? $x = 4$, find y.

$y = -16x^2 + 64x + 80$

a) axis $= \frac{-b}{2a} = \frac{-64}{2(-16)} = \frac{-64}{-32} = 2$ $y = -16(2)^2 + 64(2) + 80 = 144$

max height of 144 ft at 2 sec.

b) $y = -16(x^2 - 4x - 5) = -16(x - 5)(x + 1)$ roots @ $x = -1, 5$

hits ground after 5 sec.

c) $108 = -16x^2 + 64x + 80$ $x = 3.5$ sec & $\frac{1}{2}$ sec. } from graph calc

d) $128 = -16x^2 + 64x + 80$ $x = 3$ sec & 1 sec }

e) $y = -16(4)^2 + 64(4) + 80$

$y = 80$ ft.