1

Functions and Their Graphs

In Mathematics

Functions show how one variable is related to another variable.

In Real Life

Functions are used to estimate values, simulate processes, and discover relationships. For instance, you can model the enrollment rate of children in preschool and estimate the year in which the rate will reach a certain number. Such an estimate can be used to plan measures for meeting future needs, such as hiring additional teachers and buying more books. (See Exercise 113, page 64.)

Jose Luis Pelaez/Getty Images

IN CAREERS

There are many careers that use functions. Several are listed below.

- Financial analyst
  Exercise 95, page 51
- Biologist
  Exercise 73, page 91
- Tax preparer
  Example 3, page 104
- Oceanographer
  Exercise 83, page 112

1.1 RECTANGULAR COORDINATES

What you should learn

- Plot points in the Cartesian plane.
- Use the Distance Formula to find the distance between two points.
- Use the Midpoint Formula to find the midpoint of a line segment.
- Use a coordinate plane to model and solve real-life problems.

Why you should learn it

The Cartesian plane can be used to represent relationships between two variables. For instance, in Exercise 70 on page 11, a graph represents the minimum wage in the United States from 1950 to 2009.

The Cartesian Plane

Just as you can represent real numbers by points on a real number line, you can represent ordered pairs of real numbers by points in a plane called the **rectangular coordinate system,** or the **Cartesian plane,** named after the French mathematician René Descartes (1596–1650).

The Cartesian plane is formed by using two real number lines intersecting at right angles, as shown in Figure 1.1. The horizontal real number line is usually called the ***x*-axis,** and the vertical real number line is usually called the ***y*-axis.** The point of intersection of these two axes is the **origin,** and the two axes divide the plane into four parts called **quadrants.**

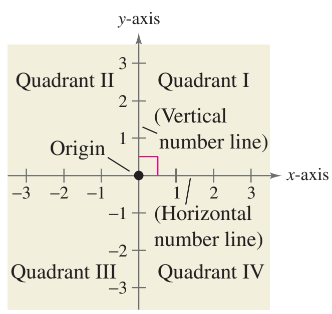


FIGURE 1.1

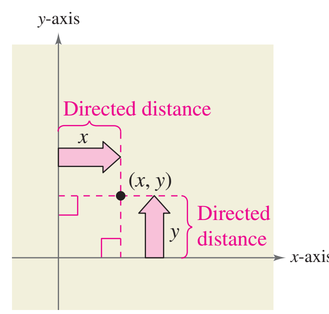
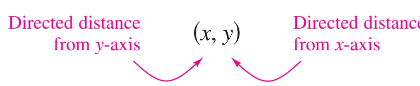


FIGURE 1.2

Each point in the plane corresponds to an **ordered pair** (x, y) of real numbers x and y, called **coordinates** of the point. The ***x*-coordinate** represents the directed distance from the y-axis to the point, and the ***y*-coordinate** represents the directed distance from the x-axis to the point, as shown in Figure 1.2.



The notation (x, y) denotes both a point in the plane and an open interval on the real number line. The context will tell you which meaning is intended.

Example 1 Plotting Points in the Cartesian Plane

Plot the points $(-1, 2)$, $(3, 4)$, $(0, 0)$, $(3, 0)$, and $(-2, -3)$.

Solution

To plot the point $(-1, 2)$, imagine a vertical line through -1 on the x-axis and a horizontal line through 2 on the y-axis. The intersection of these two lines is the point $(-1, 2)$. The other four points can be plotted in a similar way, as shown in Figure 1.3.

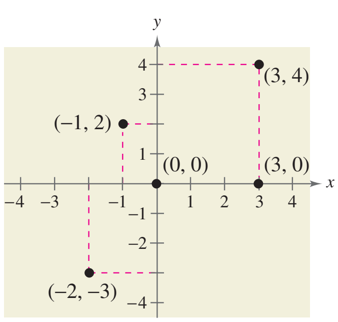


FIGURE 1.3

CHECKPoint Now try Exercise 7.

The beauty of a rectangular coordinate system is that it allows you to *see* relationships between two variables. It would be difficult to overestimate the importance of Descartes's introduction of coordinates in the plane. Today, his ideas are in common use in virtually every scientific and business-related field.

Example 2 Sketching a Scatter Plot

Year, t	Subscribers, N
1994	24.1
1995	33.8
1996	44.0
1997	55.3
1998	69.2
1999	86.0
2000	109.5
2001	128.4
2002	140.8
2003	158.7
2004	182.1
2005	207.9
2006	233.0
2007	255.4

From 1994 through 2007, the numbers N (in millions) of subscribers to a cellular telecommunication service in the United States are shown in the table, where t represents the year. Sketch a scatter plot of the data. (Source: CTIA-The Wireless Association)

Solution

To sketch a *scatter plot* of the data shown in the table, you simply represent each pair of values by an ordered pair (t, N) and plot the resulting points, as shown in Figure 1.4. For instance, the first pair of values is represented by the ordered pair (1994, 24.1). Note that the break in the t-axis indicates that the numbers between 0 and 1994 have been omitted.

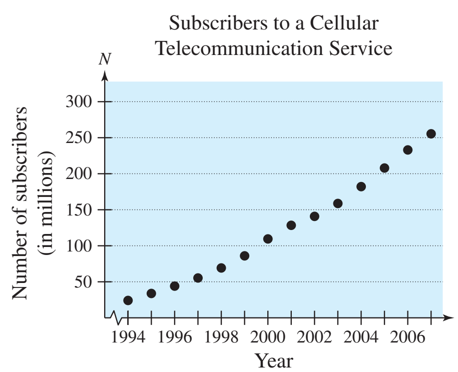


FIGURE 1.4

CHECK*Point* Now try Exercise 25.

In Example 2, you could have let $t = 1$ represent the year 1994. In that case, the horizontal axis would not have been broken, and the tick marks would have been labeled 1 through 14 (instead of 1994 through 2007).

TECHNOLOGY

The scatter plot in Example 2 is only one way to represent the data graphically. You could also represent the data using a bar graph or a line graph. If you have access to a graphing utility, try using it to represent graphically the data given in Example 2.

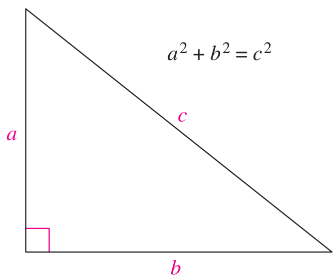


FIGURE 1.5

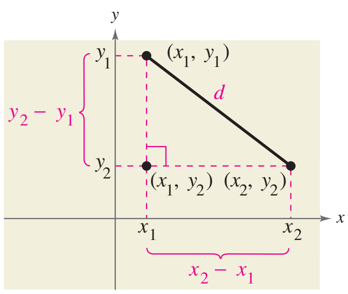


FIGURE 1.6

The Pythagorean Theorem and the Distance Formula

The following famous theorem is used extensively throughout this course.

Pythagorean Theorem

For a right triangle with hypotenuse of length c and sides of lengths a and b, you have $a^2 + b^2 = c^2$, as shown in Figure 1.5. (The converse is also true. That is, if $a^2 + b^2 = c^2$, then the triangle is a right triangle.)

Suppose you want to determine the distance d between two points (x_1, y_1) and (x_2, y_2) in the plane. With these two points, a right triangle can be formed, as shown in Figure 1.6. The length of the vertical side of the triangle is $|y_2 - y_1|$, and the length of the horizontal side is $|x_2 - x_1|$. By the Pythagorean Theorem, you can write

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This result is the **Distance Formula.**

The Distance Formula

The distance d between the points (x_1, y_1) and (x_2, y_2) in the plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Example 3 Finding a Distance

Find the distance between the points $(-2, 1)$ and $(3, 4)$.

Algebraic Solution

Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (3, 4)$. Then apply the Distance Formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$= \sqrt{[3 - (-2)]^2 + (4 - 1)^2} \quad \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.$$

$$= \sqrt{(5)^2 + (3)^2} \quad \text{Simplify.}$$

$$= \sqrt{34} \quad \text{Simplify.}$$

$$\approx 5.83 \quad \text{Use a calculator.}$$

So, the distance between the points is about 5.83 units. You can use the Pythagorean Theorem to check that the distance is correct.

$$d^2 \stackrel{?}{=} 3^2 + 5^2 \quad \text{Pythagorean Theorem}$$

$$(\sqrt{34})^2 \stackrel{?}{=} 3^2 + 5^2 \quad \text{Substitute for } d.$$

$$34 = 34 \quad \text{Distance checks. } \checkmark$$

CHECKPoint Now try Exercise 31.

Graphical Solution

Use centimeter graph paper to plot the points $A(-2, 1)$ and $B(3, 4)$. Carefully sketch the line segment from A to B. Then use a centimeter ruler to measure the length of the segment.

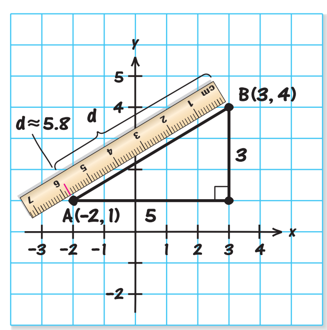


FIGURE 1.7

The line segment measures about 5.8 centimeters, as shown in Figure 1.7. So, the distance between the points is about 5.8 units.

Example 4 Verifying a Right Triangle

Show that the points (2, 1), (4, 0), and (5, 7) are vertices of a right triangle.

Solution

The three points are plotted in Figure 1.8. Using the Distance Formula, you can find the lengths of the three sides as follows.

$$d_1 = \sqrt{(5-2)^2 + (7-1)^2} = \sqrt{9+36} = \sqrt{45}$$
$$d_2 = \sqrt{(4-2)^2 + (0-1)^2} = \sqrt{4+1} = \sqrt{5}$$
$$d_3 = \sqrt{(5-4)^2 + (7-0)^2} = \sqrt{1+49} = \sqrt{50}$$

Because

$$(d_1)^2 + (d_2)^2 = 45 + 5 = 50 = (d_3)^2$$

you can conclude by the Pythagorean Theorem that the triangle must be a right triangle.

CHECKPoint Now try Exercise 43.

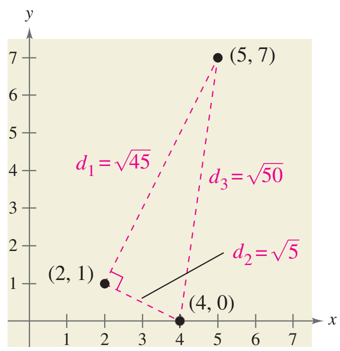


FIGURE 1.8

Algebra Help

You can review the techniques for evaluating a radical in Appendix A.2.

The Midpoint Formula

To find the **midpoint** of the line segment that joins two points in a coordinate plane, you can simply find the average values of the respective coordinates of the two endpoints using the **Midpoint Formula.**

> **The Midpoint Formula**
>
> The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by the Midpoint Formula
>
> $$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

For a proof of the Midpoint Formula, see Proofs in Mathematics on page 122.

Example 5 Finding a Line Segment's Midpoint

Find the midpoint of the line segment joining the points $(-5, -3)$ and $(9, 3)$.

Solution

Let $(x_1, y_1) = (-5, -3)$ and $(x_2, y_2) = (9, 3)$.

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \qquad \text{Midpoint Formula}$$
$$= \left(\frac{-5 + 9}{2}, \frac{-3 + 3}{2}\right) \qquad \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.$$
$$= (2, 0) \qquad \text{Simplify.}$$

The midpoint of the line segment is (2, 0), as shown in Figure 1.9.

CHECKPoint Now try Exercise 47(c).

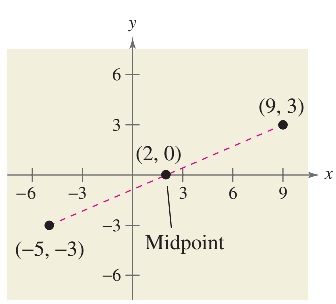


FIGURE 1.9

Applications

Example 6 Finding the Length of a Pass

A football quarterback throws a pass from the 28-yard line, 40 yards from the sideline. The pass is caught by a wide receiver on the 5-yard line, 20 yards from the same sideline, as shown in Figure 1.10. How long is the pass?

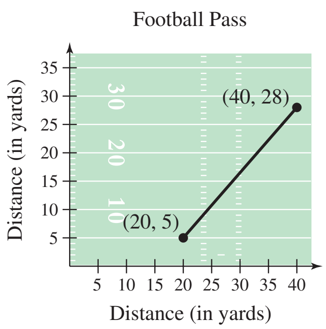


FIGURE 1.10

Solution

You can find the length of the pass by finding the distance between the points (40, 28) and (20, 5).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{Distance Formula}$$

$$= \sqrt{(40 - 20)^2 + (28 - 5)^2} \qquad \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.$$

$$= \sqrt{400 + 529} \qquad \text{Simplify.}$$

$$= \sqrt{929} \qquad \text{Simplify.}$$

$$\approx 30 \qquad \text{Use a calculator.}$$

So, the pass is about 30 yards long.

CHECKPoint Now try Exercise 57.

In Example 6, the scale along the goal line does not normally appear on a football field. However, when you use coordinate geometry to solve real-life problems, you are free to place the coordinate system in any way that is convenient for the solution of the problem.

Example 7 Estimating Annual Revenue

Barnes & Noble had annual sales of approximately \$5.1 billion in 2005, and \$5.4 billion in 2007. Without knowing any additional information, what would you estimate the 2006 sales to have been? (Source: Barnes & Noble, Inc.)

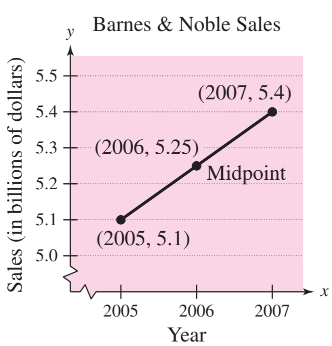


FIGURE 1.11

Solution

One solution to the problem is to assume that sales followed a linear pattern. With this assumption, you can estimate the 2006 sales by finding the midpoint of the line segment connecting the points (2005, 5.1) and (2007, 5.4).

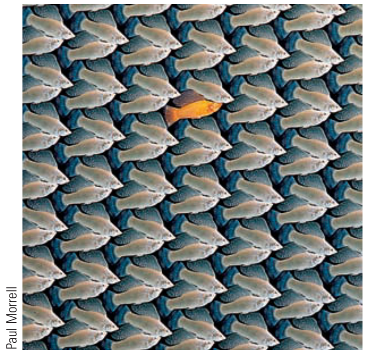
$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \qquad \text{Midpoint Formula}$$

$$= \left(\frac{2005 + 2007}{2}, \frac{5.1 + 5.4}{2}\right) \qquad \text{Substitute for } x_1, x_2, y_1 \text{ and } y_2.$$

$$= (2006, 5.25) \qquad \text{Simplify.}$$

So, you would estimate the 2006 sales to have been about \$5.25 billion, as shown in Figure 1.11. (The actual 2006 sales were about \$5.26 billion.)

CHECKPoint Now try Exercise 59.



Much of computer graphics, including this computer-generated goldfish tessellation, consists of transformations of points in a coordinate plane. One type of transformation, a translation, is illustrated in Example 8. Other types include reflections, rotations, and stretches.

Example 8 Translating Points in the Plane

The triangle in Figure 1.12 has vertices at the points $(-1, 2)$, $(1, -4)$, and $(2, 3)$. Shift the triangle three units to the right and two units upward and find the vertices of the shifted triangle, as shown in Figure 1.13.

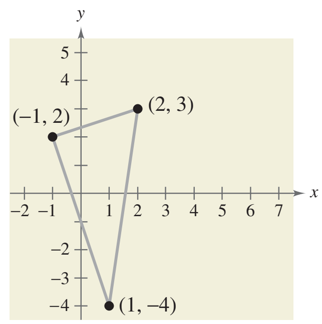


FIGURE 1.12

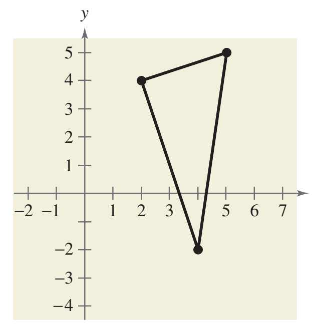


FIGURE 1.13

Solution

To shift the vertices three units to the right, add 3 to each of the x-coordinates. To shift the vertices two units upward, add 2 to each of the y-coordinates.

Original Point	*Translated Point*
$(-1, 2)$	$(-1 + 3, 2 + 2) = (2, 4)$
$(1, -4)$	$(1 + 3, -4 + 2) = (4, -2)$
$(2, 3)$	$(2 + 3, 3 + 2) = (5, 5)$

CHECK*Point* Now try Exercise 61.

The figures provided with Example 8 were not really essential to the solution. Nevertheless, it is strongly recommended that you develop the habit of including sketches with your solutions—even if they are not required.

Classroom Discussion

Extending the Example Example 8 shows how to translate points in a coordinate plane. Write a short paragraph describing how each of the following transformed points is related to the original point.

Original Point	*Transformed Point*
(x, y)	$(-x, y)$
(x, y)	$(x, -y)$
(x, y)	$(-x, -y)$

1.1 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY

1. Match each term with its definition.

(a) x-axis (i) point of intersection of vertical axis and horizontal axis

(b) y-axis (ii) directed distance from the x-axis

(c) origin (iii) directed distance from the y-axis

(d) quadrants (iv) four regions of the coordinate plane

(e) x-coordinate (v) horizontal real number line

(f) y-coordinate (vi) vertical real number line

In Exercises 2–4, fill in the blanks.

2. An ordered pair of real numbers can be represented in a plane called the rectangular coordinate system or the ________ plane.

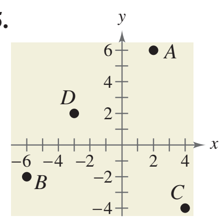
3. The ________ ________ is a result derived from the Pythagorean Theorem.

4. Finding the average values of the representative coordinates of the two endpoints of a line segment in a coordinate plane is also known as using the ________ ________.

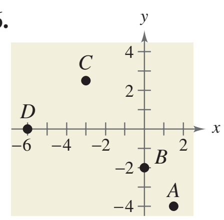
SKILLS AND APPLICATIONS

In Exercises 5 and 6, approximate the coordinates of the points.

5.



6.



In Exercises 7–10, plot the points in the Cartesian plane.

7. $(-4, 2), (-3, -6), (0, 5), (1, -4)$

8. $(0, 0), (3, 1), (-2, 4), (1, -1)$

9. $(3, 8), (0.5, -1), (5, -6), (-2, 2.5)$

10. $\left(1, -\frac{1}{3}\right), \left(\frac{3}{4}, 3\right), (-3, 4), \left(-\frac{4}{3}, -\frac{3}{2}\right)$

In Exercises 11–14, find the coordinates of the point.

11. The point is located three units to the left of the y-axis and four units above the x-axis.

12. The point is located eight units below the x-axis and four units to the right of the y-axis.

13. The point is located five units below the x-axis and the coordinates of the point are equal.

14. The point is on the x-axis and 12 units to the left of the y-axis.

In Exercises 15–24, determine the quadrant(s) in which (x, y) is located so that the condition(s) is (are) satisfied.

15. $x > 0$ and $y < 0$

16. $x < 0$ and $y < 0$

17. $x = -4$ and $y > 0$

18. $x > 2$ and $y = 3$

19. $y < -5$

20. $x > 4$

21. $x < 0$ and $-y > 0$

22. $-x > 0$ and $y < 0$

23. $xy > 0$

24. $xy < 0$

In Exercises 25 and 26, sketch a scatter plot of the data shown in the table.

25. NUMBER OF STORES The table shows the number y of Wal-Mart stores for each year x from 2000 through 2007. (Source: Wal-Mart Stores, Inc.)

Year, x	Number of stores, y
2000	4189
2001	4414
2002	4688
2003	4906
2004	5289
2005	6141
2006	6779
2007	7262

26. METEOROLOGY The table shows the lowest temperature on record y (in degrees Fahrenheit) in Duluth, Minnesota for each month x, where $x = 1$ represents January. (Source: NOAA)

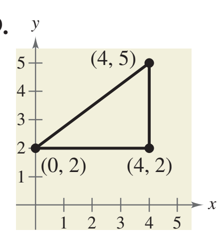
Month, x	Temperature, y
1	−39
2	−39
3	−29
4	−5
5	17
6	27
7	35
8	32
9	22
10	8
11	−23
12	−34

In Exercises 27–38, find the distance between the points.

27. $(6, -3), (6, 5)$
28. $(1, 4), (8, 4)$
29. $(-3, -1), (2, -1)$
30. $(-3, -4), (-3, 6)$
31. $(-2, 6), (3, -6)$
32. $(8, 5), (0, 20)$
33. $(1, 4), (-5, -1)$
34. $(1, 3), (3, -2)$
35. $\left(\frac{1}{2}, \frac{4}{3}\right), (2, -1)$
36. $\left(-\frac{2}{3}, 3\right), \left(-1, \frac{5}{4}\right)$
37. $(-4.2, 3.1), (-12.5, 4.8)$
38. $(9.5, -2.6), (-3.9, 8.2)$

In Exercises 39–42, (a) find the length of each side of the right triangle, and (b) show that these lengths satisfy the Pythagorean Theorem.

39.



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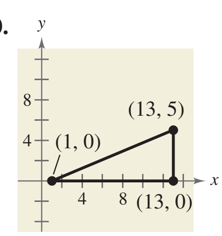
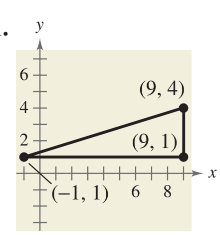
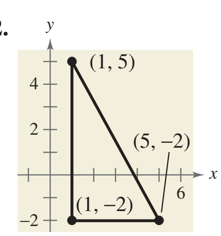


41.



42.



In Exercises 43–46, show that the points form the vertices of the indicated polygon.

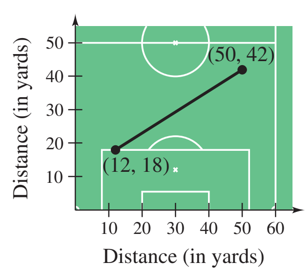
43. Right triangle: $(4, 0), (2, 1), (-1, -5)$
44. Right triangle: $(-1, 3), (3, 5), (5, 1)$
45. Isosceles triangle: $(1, -3), (3, 2), (-2, 4)$
46. Isosceles triangle: $(2, 3), (4, 9), (-2, 7)$

In Exercises 47–56, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.

47. $(1, 1), (9, 7)$
48. $(1, 12), (6, 0)$
49. $(-4, 10), (4, -5)$
50. $(-7, -4), (2, 8)$
51. $(-1, 2), (5, 4)$
52. $(2, 10), (10, 2)$
53. $\left(\frac{1}{2}, 1\right), \left(-\frac{5}{2}, \frac{4}{3}\right)$
54. $\left(-\frac{1}{3}, -\frac{1}{3}\right), \left(-\frac{1}{6}, -\frac{1}{2}\right)$
55. $(6.2, 5.4), (-3.7, 1.8)$
56. $(-16.8, 12.3), (5.6, 4.9)$

57. FLYING DISTANCE An airplane flies from Naples, Italy in a straight line to Rome, Italy, which is 120 kilometers north and 150 kilometers west of Naples. How far does the plane fly?

58. SPORTS A soccer player passes the ball from a point that is 18 yards from the endline and 12 yards from the sideline. The pass is received by a teammate who is 42 yards from the same endline and 50 yards from the same sideline, as shown in the figure. How long is the pass?



SALES In Exercises 59 and 60, use the Midpoint Formula to estimate the sales of Big Lots, Inc. and Dollar Tree Stores, Inc. in 2005, given the sales in 2003 and 2007. Assume that the sales followed a linear pattern. (Source: Big Lots, Inc.; Dollar Tree Stores, Inc.)

59. Big Lots

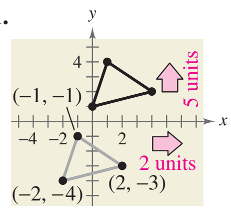
Year	Sales (in millions)
2003	\$4174
2007	\$4656

60. Dollar Tree

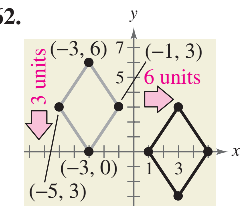
Year	Sales (in millions)
2003	$2800
2007	$4243

In Exercises 61–64, the polygon is shifted to a new position in the plane. Find the coordinates of the vertices of the polygon in its new position.

61.



62.



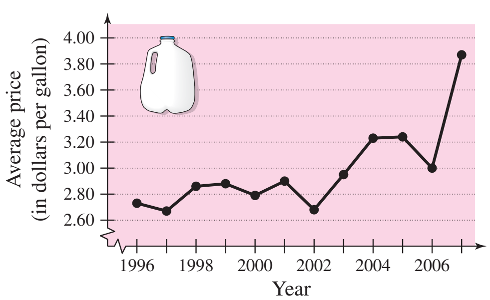
63. Original coordinates of vertices: $(-7, -2)$,$(-2, 2)$, $(-2, -4)$, $(-7, -4)$

Shift: eight units upward, four units to the right

64. Original coordinates of vertices: $(5, 8)$, $(3, 6)$, $(7, 6)$, $(5, 2)$

Shift: 6 units downward, 10 units to the left

RETAIL PRICE In Exercises 65 and 66, use the graph, which shows the average retail prices of 1 gallon of whole milk from 1996 to 2007. (Source: U.S. Bureau of Labor Statistics)



65. Approximate the highest price of a gallon of whole milk shown in the graph. When did this occur?

66. Approximate the percent change in the price of milk from the price in 1996 to the highest price shown in the graph.

67. ADVERTISING The graph shows the average costs of a 30-second television spot (in thousands of dollars) during the Super Bowl from 2000 to 2008. (Source: Nielson Media and TNS Media Intelligence)

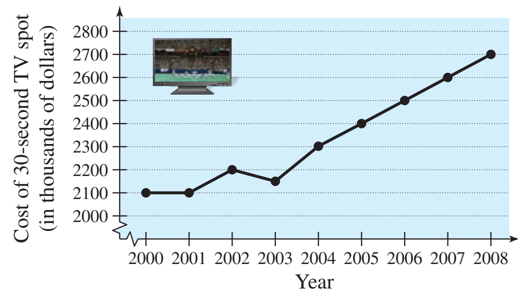
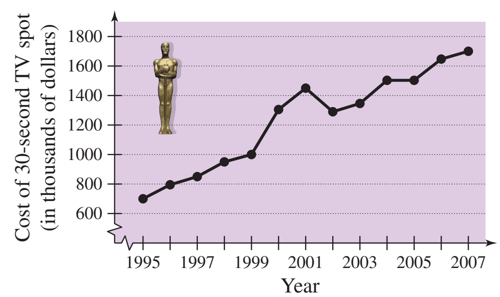


FIGURE FOR **67**

(a) Estimate the percent increase in the average cost of a 30-second spot from Super Bowl XXXIV in 2000 to Super Bowl XXXVIII in 2004.

(b) Estimate the percent increase in the average cost of a 30-second spot from Super Bowl XXXIV in 2000 to Super Bowl XLII in 2008.

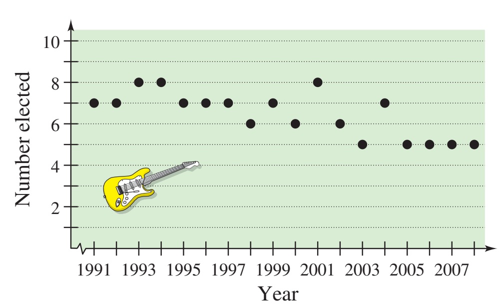
68. ADVERTISING The graph shows the average costs of a 30-second television spot (in thousands of dollars) during the Academy Awards from 1995 to 2007. (Source: Nielson Monitor-Plus)



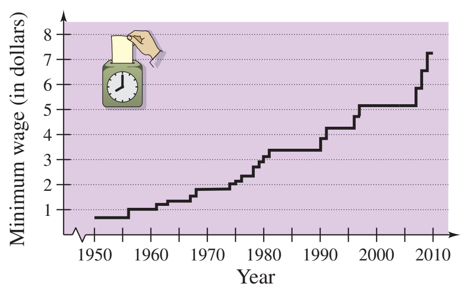
(a) Estimate the percent increase in the average cost of a 30-second spot in 1996 to the cost in 2002.

(b) Estimate the percent increase in the average cost of a 30-second spot in 1996 to the cost in 2007.

69. MUSIC The graph shows the numbers of performers who were elected to the Rock and Roll Hall of Fame from 1991 through 2008. Describe any trends in the data. From these trends, predict the number of performers elected in 2010. (Source: rockhall.com)



70. LABOR FORCE Use the graph below, which shows the minimum wage in the United States (in dollars) from 1950 to 2009. (Source: U.S. Department of Labor)



(a) Which decade shows the greatest increase in minimum wage?

(b) Approximate the percent increases in the minimum wage from 1990 to 1995 and from 1995 to 2009.

(c) Use the percent increase from 1995 to 2009 to predict the minimum wage in 2013.

(d) Do you believe that your prediction in part (c) is reasonable? Explain.

71. SALES The Coca-Cola Company had sales of $19,805 million in 1999 and $28,857 million in 2007. Use the Midpoint Formula to estimate the sales in 2003. Assume that the sales followed a linear pattern. (Source: The Coca-Cola Company)

72. DATA ANALYSIS: EXAM SCORES The table shows the mathematics entrance test scores x and the final examination scores y in an algebra course for a sample of 10 students.

x	22	29	35	40	44	48	53	58	65	76
y	53	74	57	66	79	90	76	93	83	99

(a) Sketch a scatter plot of the data.

(b) Find the entrance test score of any student with a final exam score in the 80s.

(c) Does a higher entrance test score imply a higher final exam score? Explain.

73. DATA ANALYSIS: MAIL The table shows the number y of pieces of mail handled (in billions) by the U.S. Postal Service for each year x from 1996 through 2008. (Source: U.S. Postal Service)

Year, x	Pieces of mail, y
1996	183
1997	191
1998	197
1999	202
2000	208
2001	207
2002	203
2003	202
2004	206
2005	212
2006	213
2007	212
2008	203

TABLE FOR 73

(a) Sketch a scatter plot of the data.

(b) Approximate the year in which there was the greatest decrease in the number of pieces of mail handled.

(c) Why do you think the number of pieces of mail handled decreased?

74. DATA ANALYSIS: ATHLETICS The table shows the numbers of men's M and women's W college basketball teams for each year x from 1994 through 2007. (Source: National Collegiate Athletic Association)

Year, x	Men's teams, M	Women's teams, W
1994	858	859
1995	868	864
1996	866	874
1997	865	879
1998	895	911
1999	926	940
2000	932	956
2001	937	958
2002	936	975
2003	967	1009
2004	981	1008
2005	983	1036
2006	984	1018
2007	982	1003

(a) Sketch scatter plots of these two sets of data on the same set of coordinate axes.

(b) Find the year in which the numbers of men's and women's teams were nearly equal.

(c) Find the year in which the difference between the numbers of men's and women's teams was the greatest. What was this difference?

EXPLORATION

75. A line segment has (x_1, y_1) as one endpoint and (x_m, y_m) as its midpoint. Find the other endpoint (x_2, y_2) of the line segment in terms of x_1, y_1, x_m, and y_m.

76. Use the result of Exercise 75 to find the coordinates of the endpoint of a line segment if the coordinates of the other endpoint and midpoint are, respectively,

(a) $(1, -2), (4, -1)$ and (b) $(-5, 11), (2, 4)$.

77. Use the Midpoint Formula three times to find the three points that divide the line segment joining (x_1, y_1) and (x_2, y_2) into four parts.

78. Use the result of Exercise 77 to find the points that divide the line segment joining the given points into four equal parts.

(a) $(1, -2), (4, -1)$ (b) $(-2, -3), (0, 0)$

79. MAKE A CONJECTURE Plot the points $(2, 1)$, $(-3, 5)$, and $(7, -3)$ on a rectangular coordinate system. Then change the sign of the x-coordinate of each point and plot the three new points on the same rectangular coordinate system. Make a conjecture about the location of a point when each of the following occurs.

(a) The sign of the x-coordinate is changed.

(b) The sign of the y-coordinate is changed.

(c) The signs of both the x- and y-coordinates are changed.

80. COLLINEAR POINTS Three or more points are *collinear* if they all lie on the same line. Use the steps below to determine if the set of points $\{A(2, 3), B(2, 6), C(6, 3)\}$ and the set of points $\{A(8, 3), B(5, 2), C(2, 1)\}$ are collinear.

(a) For each set of points, use the Distance Formula to find the distances from A to B, from B to C, and from A to C. What relationship exists among these distances for each set of points?

(b) Plot each set of points in the Cartesian plane. Do all the points of either set appear to lie on the same line?

(c) Compare your conclusions from part (a) with the conclusions you made from the graphs in part (b). Make a general statement about how to use the Distance Formula to determine collinearity.

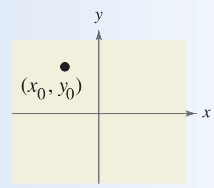
TRUE OR FALSE? In Exercises 81 and 82, determine whether the statement is true or false. Justify your answer.

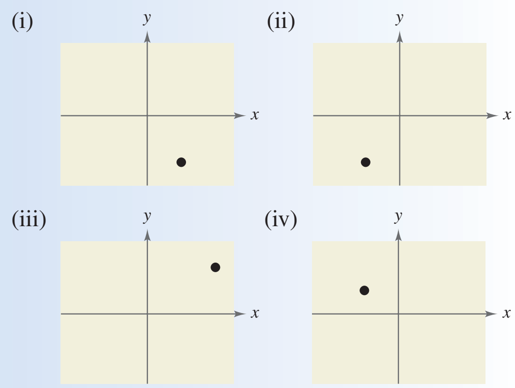
81. In order to divide a line segment into 16 equal parts, you would have to use the Midpoint Formula 16 times.

82. The points $(-8, 4)$, $(2, 11)$, and $(-5, 1)$ represent the vertices of an isosceles triangle.

83. THINK ABOUT IT When plotting points on the rectangular coordinate system, is it true that the scales on the x- and y-axes must be the same? Explain.

84. CAPSTONE Use the plot of the point (x_0, y_0) in the figure. Match the transformation of the point with the correct plot. Explain your reasoning. [The plots are labeled (i), (ii), (iii), and (iv).]





(a) $(x_0, -y_0)$ (b) $(-2x_0, y_0)$

(c) $\left(x_0, \frac{1}{2}y_0\right)$ (d) $(-x_0, -y_0)$

85. PROOF Prove that the diagonals of the parallelogram in the figure intersect at their midpoints.