1.2 GRAPHS OF EQUATIONS

What you should learn

- Sketch graphs of equations.
- Find x- and y-intercepts of graphs of equations.
- Use symmetry to sketch graphs of equations.
- Find equations of and sketch graphs of circles.
- Use graphs of equations in solving real-life problems.

Why you should learn it

The graph of an equation can help you see relationships between real-life quantities. For example, in Exercise 87 on page 23, a graph can be used to estimate the life expectancies of children who are born in 2015.

The Graph of an Equation

In Section 1.1, you used a coordinate system to represent graphically the relationship between two quantities. There, the graphical picture consisted of a collection of points in a coordinate plane.

Frequently, a relationship between two quantities is expressed as an **equation in two variables.** For instance, $y = 7 - 3x$ is an equation in x and y. An ordered pair (a, b) is a **solution** or **solution point** of an equation in x and y if the equation is true when a is substituted for x and b is substituted for y. For instance, $(1, 4)$ is a solution of $y = 7 - 3x$ because $4 = 7 - 3(1)$ is a true statement.

In this section you will review some basic procedures for sketching the graph of an equation in two variables. The **graph of an equation** is the set of all points that are solutions of the equation.

Example 1 Determining Solution Points

Determine whether (a) $(2, 13)$ and (b) $(-1, -3)$ lie on the graph of $y = 10x - 7$.

Solution

a. $y = 10x - 7$ Write original equation.

$13 \stackrel{?}{=} 10(2) - 7$ Substitute 2 for x and 13 for y.

$13 = 13$ $(2, 13)$ is a solution. ✓

The point $(2, 13)$ *does* lie on the graph of $y = 10x - 7$ because it is a solution point of the equation.

b. $y = 10x - 7$ Write original equation.

$-3 \stackrel{?}{=} 10(-1) - 7$ Substitute -1 for x and -3 for y.

$-3 \neq -17$ $(-1, -3)$ is not a solution.

The point $(-1, -3)$ *does not* lie on the graph of $y = 10x - 7$ because it is *not* a solution point of the equation.

CHECK*Point* Now try Exercise 7.

The basic technique used for sketching the graph of an equation is the **point-plotting method.**

Algebra Help

When evaluating an expression or an equation, remember to follow the Basic Rules of Algebra. To review these rules, see Appendix A.1.

Sketching the Graph of an Equation by Point Plotting

1. If possible, rewrite the equation so that one of the variables is isolated on one side of the equation.
2. Make a table of values showing several solution points.
3. Plot these points on a rectangular coordinate system.
4. Connect the points with a smooth curve or line.

Example 2 Sketching the Graph of an Equation

Sketch the graph of

$$y = 7 - 3x.$$

Solution

Because the equation is already solved for y, construct a table of values that consists of several solution points of the equation. For instance, when $x = -1$,

$$y = 7 - 3(-1)$$
$$= 10$$

which implies that $(-1, 10)$ is a solution point of the graph.

x	$y = 7 - 3x$	(x, y)
-1	10	$(-1, 10)$
0	7	$(0, 7)$
1	4	$(1, 4)$
2	1	$(2, 1)$
3	-2	$(3, -2)$
4	-5	$(4, -5)$

From the table, it follows that

$$(-1, 10), (0, 7), (1, 4), (2, 1), (3, -2), \text{ and } (4, -5)$$

are solution points of the equation. After plotting these points, you can see that they appear to lie on a line, as shown in Figure 1.14. The graph of the equation is the line that passes through the six plotted points.

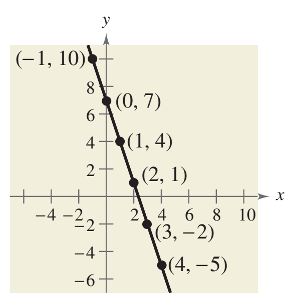


FIGURE 1.14

CHECKPoint Now try Exercise 15.

Example 3 Sketching the Graph of an Equation

Sketch the graph of

$$y = x^2 - 2.$$

Solution

Because the equation is already solved for y, begin by constructing a table of values.

x	-2	-1	0	1	2	3
$y = x^2 - 2$	2	-1	-2	-1	2	7
(x, y)	$(-2, 2)$	$(-1, -1)$	$(0, -2)$	$(1, -1)$	$(2, 2)$	$(3, 7)$

Next, plot the points given in the table, as shown in Figure 1.15. Finally, connect the points with a smooth curve, as shown in Figure 1.16.

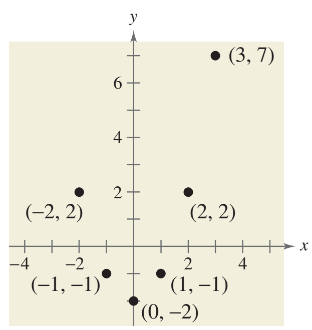


FIGURE 1.15

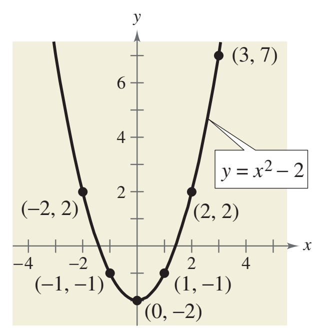


FIGURE 1.16

CHECKPoint Now try Exercise 17.

Study Tip

One of your goals in this course is to learn to classify the basic shape of a graph from its equation. For instance, you will learn that the *linear equation* in Example 2 has the form

$$y = mx + b$$

and its graph is a line. Similarly, the *quadratic equation* in Example 3 has the form

$$y = ax^2 + bx + c$$

and its graph is a parabola.

The point-plotting method demonstrated in Examples 2 and 3 is easy to use, but it has some shortcomings. With too few solution points, you can misrepresent the graph of an equation. For instance, if only the four points

$$(-2, 2), (-1, -1), (1, -1), \text{ and } (2, 2)$$

in Figure 1.15 were plotted, any one of the three graphs in Figure 1.17 would be reasonable.

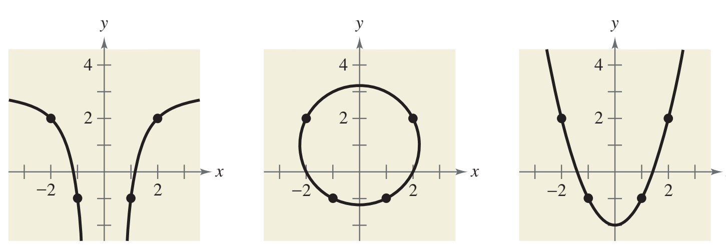
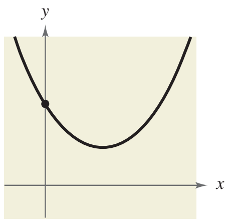
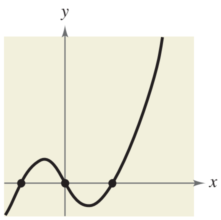


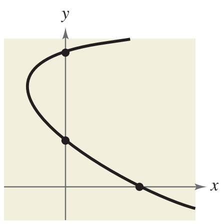
FIGURE 1.17



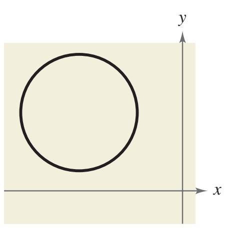
No x-intercepts; one y-intercept



Three x-intercepts; one y-intercept



One x-intercept; two y-intercepts



No intercepts

FIGURE 1.18

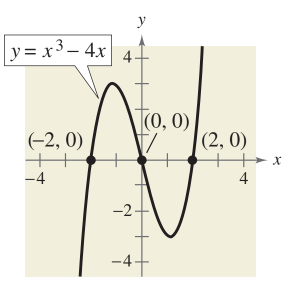


FIGURE 1.19

TECHNOLOGY

To graph an equation involving x and y on a graphing utility, use the following procedure.

1. Rewrite the equation so that y is isolated on the left side.
2. Enter the equation into the graphing utility.
3. Determine a *viewing window* that shows all important features of the graph.
4. Graph the equation.

Intercepts of a Graph

It is often easy to determine the solution points that have zero as either the x-coordinate or the y-coordinate. These points are called **intercepts** because they are the points at which the graph intersects or touches the x- or y-axis. It is possible for a graph to have no intercepts, one intercept, or several intercepts, as shown in Figure 1.18.

Note that an x-intercept can be written as the ordered pair $(x, 0)$ and a y-intercept can be written as the ordered pair $(0, y)$. Some texts denote the x-intercept as the x-coordinate of the point $(a, 0)$ [and the y-intercept as the y-coordinate of the point $(0, b)$] rather than the point itself. Unless it is necessary to make a distinction, we will use the term *intercept* to mean either the point or the coordinate.

Finding Intercepts

1. To find x-intercepts, let y be zero and solve the equation for x.
2. To find y-intercepts, let x be zero and solve the equation for y.

Example 4 Finding x- and y-Intercepts

Find the x- and y-intercepts of the graph of $y = x^3 - 4x$.

Solution

Let $y = 0$. Then

$$0 = x^3 - 4x = x(x^2 - 4)$$

has solutions $x = 0$ and $x = \pm 2$.

x-intercepts: $(0, 0), (2, 0), (-2, 0)$

Let $x = 0$. Then

$$y = (0)^3 - 4(0)$$

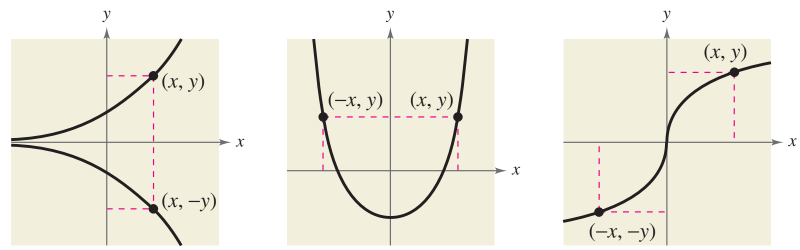
has one solution, $y = 0$.

y-intercept: $(0, 0)$ See Figure 1.19.

CHECKPoint Now try Exercise 23.

Symmetry

Graphs of equations can have **symmetry** with respect to one of the coordinate axes or with respect to the origin. Symmetry with respect to the x-axis means that if the Cartesian plane were folded along the x-axis, the portion of the graph above the x-axis would coincide with the portion below the x-axis. Symmetry with respect to the y-axis or the origin can be described in a similar manner, as shown in Figure 1.20.



x-axis symmetry y-axis symmetry Origin symmetry
FIGURE 1.20

Knowing the symmetry of a graph *before* attempting to sketch it is helpful, because then you need only half as many solution points to sketch the graph. There are three basic types of symmetry, described as follows.

Graphical Tests for Symmetry

1. A graph is **symmetric with respect to the x-axis** if, whenever (x, y) is on the graph, $(x, -y)$ is also on the graph.
2. A graph is **symmetric with respect to the y-axis** if, whenever (x, y) is on the graph, $(-x, y)$ is also on the graph.
3. A graph is **symmetric with respect to the origin** if, whenever (x, y) is on the graph, $(-x, -y)$ is also on the graph.

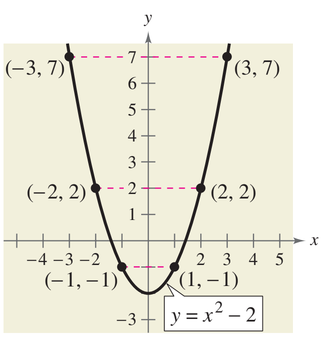


FIGURE 1.21 y-axis symmetry

You can conclude that the graph of $y = x^2 - 2$ is symmetric with respect to the y-axis because the point $(-x, y)$ is also on the graph of $y = x^2 - 2$. (See the table below and Figure 1.21.)

x	-3	-2	-1	1	2	3
y	7	2	-1	-1	2	7
(x, y)	$(-3, 7)$	$(-2, 2)$	$(-1, -1)$	$(1, -1)$	$(2, 2)$	$(3, 7)$

Algebraic Tests for Symmetry

1. The graph of an equation is symmetric with respect to the x-axis if replacing y with $-y$ yields an equivalent equation.
2. The graph of an equation is symmetric with respect to the y-axis if replacing x with $-x$ yields an equivalent equation.
3. The graph of an equation is symmetric with respect to the origin if replacing x with $-x$ and y with $-y$ yields an equivalent equation.

Example 5 Testing for Symmetry

Test $y = 2x^3$ for symmetry with respect to both axes and the origin.

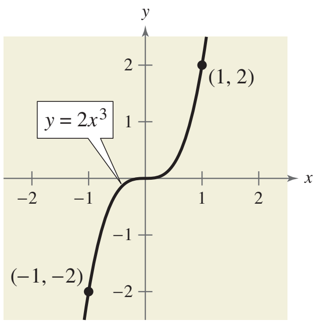


FIGURE 1.22

Solution

x-axis:	$y = 2x^3$	Write original equation.
	$-y = 2x^3$	Replace y with $-y$. Result is *not* an equivalent equation.
y-axis:	$y = 2x^3$	Write original equation.
	$y = 2(-x)^3$	Replace x with $-x$.
	$y = -2x^3$	Simplify. Result is *not* an equivalent equation.
Origin:	$y = 2x^3$	Write original equation.
	$-y = 2(-x)^3$	Replace y with $-y$ and x with $-x$.
	$-y = -2x^3$	Simplify.
	$y = 2x^3$	Equivalent equation

Of the three tests for symmetry, the only one that is satisfied is the test for origin symmetry (see Figure 1.22).

CHECKPoint Now try Exercise 33.

Example 6 Using Symmetry as a Sketching Aid

Use symmetry to sketch the graph of $x - y^2 = 1$.

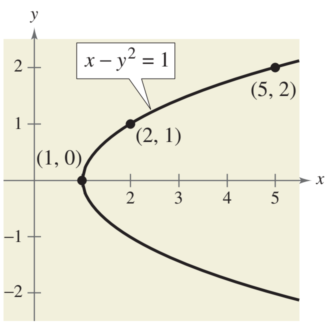


FIGURE 1.23

Solution

Of the three tests for symmetry, the only one that is satisfied is the test for x-axis symmetry because $x - (-y)^2 = 1$ is equivalent to $x - y^2 = 1$. So, the graph is symmetric with respect to the x-axis. Using symmetry, you only need to find the solution points above the x-axis and then reflect them to obtain the graph, as shown in Figure 1.23.

CHECKPoint Now try Exercise 49.

Algebra Help

In Example 7, $|x - 1|$ is an absolute value expression. You can review the techniques for evaluating an absolute value expression in Appendix A.1.

Example 7 Sketching the Graph of an Equation

Sketch the graph of $y = |x - 1|$.

Solution

This equation fails all three tests for symmetry and consequently its graph is not symmetric with respect to either axis or to the origin. The absolute value sign indicates that y is always nonnegative. Create a table of values and plot the points, as shown in Figure 1.24. From the table, you can see that $x = 0$ when $y = 1$. So, the y-intercept is $(0, 1)$. Similarly, $y = 0$ when $x = 1$. So, the x-intercept is $(1, 0)$.

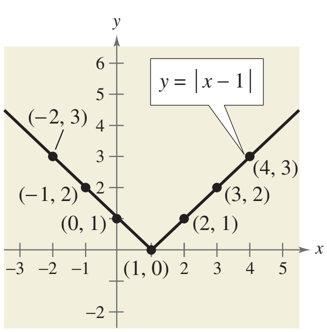


FIGURE 1.24

x	-2	-1	0	1	2	3	4
$y = \|x - 1\|$	3	2	1	0	1	2	3
(x, y)	$(-2, 3)$	$(-1, 2)$	$(0, 1)$	$(1, 0)$	$(2, 1)$	$(3, 2)$	$(4, 3)$

CHECKPoint Now try Exercise 53.

Throughout this course, you will learn to recognize several types of graphs from their equations. For instance, you will learn to recognize that the graph of a second-degree equation of the form

$$y = ax^2 + bx + c$$

is a parabola (see Example 3). The graph of a **circle** is also easy to recognize.

Circles

Consider the circle shown in Figure 1.25. A point (x, y) is on the circle if and only if its distance from the center (h, k) is r. By the Distance Formula,

$$\sqrt{(x - h)^2 + (y - k)^2} = r.$$

By squaring each side of this equation, you obtain the **standard form of the equation of a circle.**

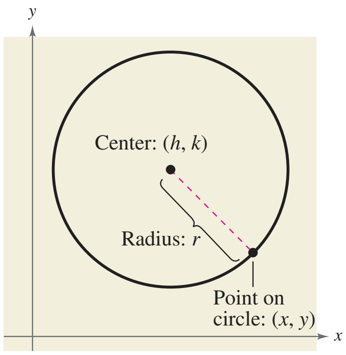


FIGURE 1.25

Standard Form of the Equation of a Circle

The point (x, y) lies on the circle of **radius** r and **center** (h, k) if and only if

$$(x - h)^2 + (y - k)^2 = r^2.$$

WARNING / CAUTION

Be careful when you are finding h and k from the standard equation of a circle. For instance, to find the correct h and k from the equation of the circle in Example 8, rewrite the quantities $(x + 1)^2$ and $(y - 2)^2$ using subtraction.

$$(x + 1)^2 = [x - (-1)]^2,$$
$$(y - 2)^2 = [y - (2)]^2$$

So, $h = -1$ and $k = 2$.

From this result, you can see that the standard form of the equation of a circle *with its center at the origin,* $(h, k) = (0, 0)$, is simply

$$x^2 + y^2 = r^2. \qquad \text{Circle with center at origin}$$

Example 8 Finding the Equation of a Circle

The point $(3, 4)$ lies on a circle whose center is at $(-1, 2)$, as shown in Figure 1.26. Write the standard form of the equation of this circle.

Solution

The radius of the circle is the distance between $(-1, 2)$ and $(3, 4)$.

$$r = \sqrt{(x - h)^2 + (y - k)^2} \qquad \text{Distance Formula}$$
$$= \sqrt{[3 - (-1)]^2 + (4 - 2)^2} \qquad \text{Substitute for } x, y, h, \text{ and } k.$$
$$= \sqrt{4^2 + 2^2} \qquad \text{Simplify.}$$
$$= \sqrt{16 + 4} \qquad \text{Simplify.}$$
$$= \sqrt{20} \qquad \text{Radius}$$

Using $(h, k) = (-1, 2)$ and $r = \sqrt{20}$, the equation of the circle is

$$(x - h)^2 + (y - k)^2 = r^2 \qquad \text{Equation of circle}$$
$$[x - (-1)]^2 + (y - 2)^2 = (\sqrt{20})^2 \qquad \text{Substitute for } h, k, \text{ and } r.$$
$$(x + 1)^2 + (y - 2)^2 = 20. \qquad \text{Standard form}$$

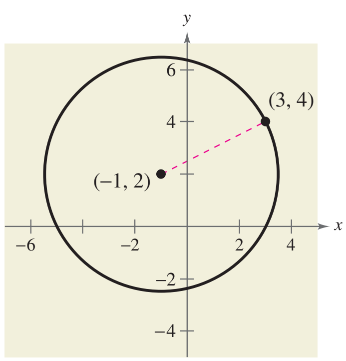


FIGURE 1.26

CHECKPoint Now try Exercise 73.

Application

In this course, you will learn that there are many ways to approach a problem. Three common approaches are illustrated in Example 9.

A *Numerical Approach:* Construct and use a table.
A *Graphical Approach:* Draw and use a graph.
An *Algebraic Approach:* Use the rules of algebra.

Study Tip

You should develop the habit of using at least two approaches to solve every problem. This helps build your intuition and helps you check that your answers are reasonable.

Example 9 Recommended Weight

The median recommended weight y (in pounds) for men of medium frame who are 25 to 59 years old can be approximated by the mathematical model

$$y = 0.073x^2 - 6.99x + 289.0, \quad 62 \le x \le 76$$

where x is the man's height (in inches). (Source: Metropolitan Life Insurance Company)

a. Construct a table of values that shows the median recommended weights for men with heights of 62, 64, 66, 68, 70, 72, 74, and 76 inches.

b. Use the table of values to sketch a graph of the model. Then use the graph to estimate *graphically* the median recommended weight for a man whose height is 71 inches.

c. Use the model to confirm *algebraically* the estimate you found in part (b).

Solution

a. You can use a calculator to complete the table, as shown at the left.

Height, x	Weight, y
62	136.2
64	140.6
66	145.6
68	151.2
70	157.4
72	164.2
74	171.5
76	179.4

b. The table of values can be used to sketch the graph of the equation, as shown in Figure 1.27. From the graph, you can estimate that a height of 71 inches corresponds to a weight of about 161 pounds.

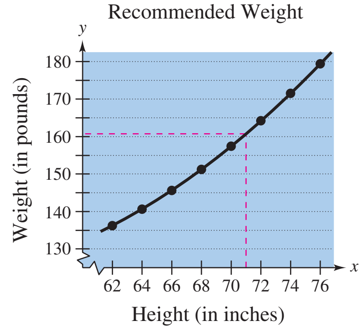


FIGURE 1.27

c. To confirm algebraically the estimate found in part (b), you can substitute 71 for x in the model.

$$y = 0.073(71)^2 - 6.99(71) + 289.0 \approx 160.70$$

So, the graphical estimate of 161 pounds is fairly good.

CHECKPoint Now try Exercise 87.

1.2 EXERCISES

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

VOCABULARY: Fill in the blanks.

1. An ordered pair (a, b) is a ________ of an equation in x and y if the equation is true when a is substituted for x, and b is substituted for y.
2. The set of all solution points of an equation is the ________ of the equation.
3. The points at which a graph intersects or touches an axis are called the ________ of the graph.
4. A graph is symmetric with respect to the ________ if, whenever (x, y) is on the graph, $(-x, y)$ is also on the graph.
5. The equation $(x - h)^2 + (y - k)^2 = r^2$ is the standard form of the equation of a ________ with center ________ and radius ________.
6. When you construct and use a table to solve a problem, you are using a ________ approach.

SKILLS AND APPLICATIONS

In Exercises 7–14, determine whether each point lies on the graph of the equation.

	Equation	*Points*	
7.	$y = \sqrt{x + 4}$	(a) $(0, 2)$	(b) $(5, 3)$
8.	$y = \sqrt{5 - x}$	(a) $(1, 2)$	(b) $(5, 0)$
9.	$y = x^2 - 3x + 2$	(a) $(2, 0)$	(b) $(-2, 8)$
10.	$y = 4 - \lvert x - 2 \rvert$	(a) $(1, 5)$	(b) $(6, 0)$
11.	$y = \lvert x - 1 \rvert + 2$	(a) $(2, 3)$	(b) $(-1, 0)$
12.	$2x - y - 3 = 0$	(a) $(1, 2)$	(b) $(1, -1)$
13.	$x^2 + y^2 = 20$	(a) $(3, -2)$	(b) $(-4, 2)$
14.	$y = \frac{1}{3}x^3 - 2x^2$	(a) $\left(2, -\frac{16}{3}\right)$	(b) $(-3, 9)$

In Exercises 15–18, complete the table. Use the resulting solution points to sketch the graph of the equation.

15. $y = -2x + 5$

x	-1	0	1	2	$\frac{5}{2}$
y					
(x, y)					

16. $y = \frac{3}{4}x - 1$

x	-2	0	1	$\frac{4}{3}$	2
y					
(x, y)					

17. $y = x^2 - 3x$

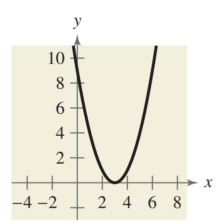
x	-1	0	1	2	3
y					
(x, y)					

18. $y = 5 - x^2$

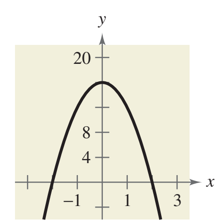
x	-2	-1	0	1	2
y					
(x, y)					

In Exercises 19–22, graphically estimate the x- and y-intercepts of the graph. Verify your results algebraically.

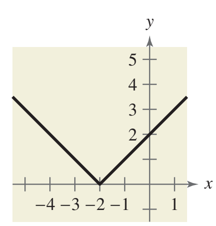
19. $y = (x - 3)^2$



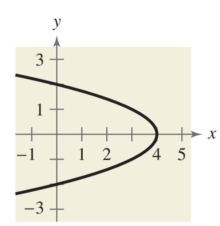
20. $y = 16 - 4x^2$



21. $y = \lvert x + 2 \rvert$



22. $y^2 = 4 - x$



In Exercises 23–32, find the x- and y-intercepts of the graph of the equation.

23. $y = 5x - 6$
24. $y = 8 - 3x$
25. $y = \sqrt{x + 4}$
26. $y = \sqrt{2x - 1}$
27. $y = \lvert 3x - 7 \rvert$
28. $y = -\lvert x + 10 \rvert$
29. $y = 2x^3 - 4x^2$
30. $y = x^4 - 25$
31. $y^2 = 6 - x$
32. $y^2 = x + 1$

In Exercises 33–40, use the algebraic tests to check for symmetry with respect to both axes and the origin.

33. $x^2 - y = 0$

34. $x - y^2 = 0$

35. $y = x^3$

36. $y = x^4 - x^2 + 3$

37. $y = \dfrac{x}{x^2 + 1}$

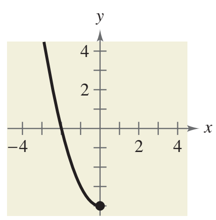
38. $y = \dfrac{1}{x^2 + 1}$

39. $xy^2 + 10 = 0$

40. $xy = 4$

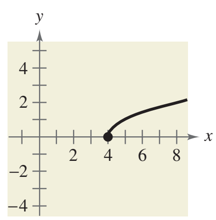
In Exercises 41–44, assume that the graph has the indicated type of symmetry. Sketch the complete graph of the equation. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.

41.



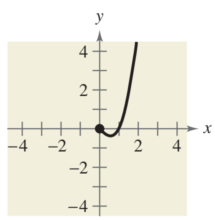
y-axis symmetry

42.



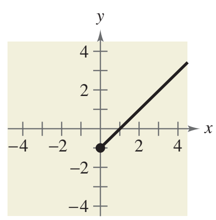
x-axis symmetry

43.



Origin symmetry

44.



y-axis symmetry

In Exercises 45–56, identify any intercepts and test for symmetry. Then sketch the graph of the equation.

45. $y = -3x + 1$

46. $y = 2x - 3$

47. $y = x^2 - 2x$

48. $y = -x^2 - 2x$

49. $y = x^3 + 3$

50. $y = x^3 - 1$

51. $y = \sqrt{x - 3}$

52. $y = \sqrt{1 - x}$

53. $y = |x - 6|$

54. $y = 1 - |x|$

55. $x = y^2 - 1$

56. $x = y^2 - 5$

In Exercises 57–68, use a graphing utility to graph the equation. Use a standard setting. Approximate any intercepts.

57. $y = 3 - \frac{1}{2}x$

58. $y = \frac{2}{3}x - 1$

59. $y = x^2 - 4x + 3$

60. $y = x^2 + x - 2$

61. $y = \dfrac{2x}{x - 1}$

62. $y = \dfrac{4}{x^2 + 1}$

63. $y = \sqrt[3]{x} + 2$

64. $y = \sqrt[3]{x + 1}$

65. $y = x\sqrt{x + 6}$

66. $y = (6 - x)\sqrt{x}$

67. $y = |x + 3|$

68. $y = 2 - |x|$

In Exercises 69–76, write the standard form of the equation of the circle with the given characteristics.

69. Center: $(0, 0)$; Radius: 4

70. Center: $(0, 0)$; Radius: 5

71. Center: $(2, -1)$; Radius: 4

72. Center: $(-7, -4)$; Radius: 7

73. Center: $(-1, 2)$; Solution point: $(0, 0)$

74. Center: $(3, -2)$; Solution point: $(-1, 1)$

75. Endpoints of a diameter: $(0, 0)$, $(6, 8)$

76. Endpoints of a diameter: $(-4, -1)$, $(4, 1)$

In Exercises 77–82, find the center and radius of the circle, and sketch its graph.

77. $x^2 + y^2 = 25$

78. $x^2 + y^2 = 16$

79. $(x - 1)^2 + (y + 3)^2 = 9$

80. $x^2 + (y - 1)^2 = 1$

81. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$

82. $(x - 2)^2 + (y + 3)^2 = \frac{16}{9}$

83. DEPRECIATION A hospital purchases a new magnetic resonance imaging (MRI) machine for \$500,000. The depreciated value y (reduced value) after t years is given by $y = 500{,}000 - 40{,}000t$, $0 \le t \le 8$. Sketch the graph of the equation.

84. CONSUMERISM You purchase an all-terrain vehicle (ATV) for \$8000. The depreciated value y after t years is given by $y = 8000 - 900t$, $0 \le t \le 6$. Sketch the graph of the equation.

85. GEOMETRY A regulation NFL playing field (including the end zones) of length x and width y has a perimeter of $346\frac{2}{3}$ or $\frac{1040}{3}$ yards.

(a) Draw a rectangle that gives a visual representation of the problem. Use the specified variables to label the sides of the rectangle.

(b) Show that the width of the rectangle is $y = \dfrac{520}{3} - x$ and its area is $A = x\left(\dfrac{520}{3} - x\right)$.

(c) Use a graphing utility to graph the area equation. Be sure to adjust your window settings.

(d) From the graph in part (c), estimate the dimensions of the rectangle that yield a maximum area.

(e) Use your school's library, the Internet, or some other reference source to find the actual dimensions and area of a regulation NFL playing field and compare your findings with the results of part (d).

The symbol indicates an exercise or a part of an exercise in which you are instructed to use a graphing utility.

86. GEOMETRY A soccer playing field of length x and width y has a perimeter of 360 meters.

(a) Draw a rectangle that gives a visual representation of the problem. Use the specified variables to label the sides of the rectangle.

(b) Show that the width of the rectangle is $y = 180 - x$ and its area is $A = x(180 - x)$.

(c) Use a graphing utility to graph the area equation. Be sure to adjust your window settings.

(d) From the graph in part (c), estimate the dimensions of the rectangle that yield a maximum area.

(e) Use your school's library, the Internet, or some other reference source to find the actual dimensions and area of a regulation Major League Soccer field and compare your findings with the results of part (d).

87. POPULATION STATISTICS The table shows the life expectancies of a child (at birth) in the United States for selected years from 1920 to 2000. (Source: U.S. National Center for Health Statistics)

Year	Life Expectancy, y
1920	54.1
1930	59.7
1940	62.9
1950	68.2
1960	69.7
1970	70.8
1980	73.7
1990	75.4
2000	77.0

A model for the life expectancy during this period is

$$y = -0.0025t^2 + 0.574t + 44.25, \quad 20 \le t \le 100$$

where y represents the life expectancy and t is the time in years, with $t = 20$ corresponding to 1920.

(a) Use a graphing utility to graph the data from the table and the model in the same viewing window. How well does the model fit the data? Explain.

(b) Determine the life expectancy in 1990 both graphically and algebraically.

(c) Use the graph to determine the year when life expectancy was approximately 76.0. Verify your answer algebraically.

(d) One projection for the life expectancy of a child born in 2015 is 78.9. How does this compare with the projection given by the model?

(e) Do you think this model can be used to predict the life expectancy of a child 50 years from now? Explain.

88. ELECTRONICS The resistance y (in ohms) of 1000 feet of solid copper wire at 68 degrees Fahrenheit can be approximated by the model

$$y = \frac{10{,}770}{x^2} - 0.37, \quad 5 \le x \le 100$$

where x is the diameter of the wire in mils (0.001 inch). (Source: American Wire Gage)

(a) Complete the table.

x	5	10	20	30	40	50
y						

x	60	70	80	90	100
y					

(b) Use the table of values in part (a) to sketch a graph of the model. Then use your graph to estimate the resistance when $x = 85.5$.

(c) Use the model to confirm algebraically the estimate you found in part (b).

(d) What can you conclude in general about the relationship between the diameter of the copper wire and the resistance?

EXPLORATION

89. THINK ABOUT IT Find a and b if the graph of $y = ax^2 + bx^3$ is symmetric with respect to (a) the y-axis and (b) the origin. (There are many correct answers.)

90. CAPSTONE Match the equation or equations with the given characteristic.

(i) $y = 3x^3 - 3x$ (ii) $y = (x + 3)^2$

(iii) $y = 3x - 3$ (iv) $y = \sqrt[3]{x}$

(v) $y = 3x^2 + 3$ (vi) $y = \sqrt{x + 3}$

(a) Symmetric with respect to the y-axis

(b) Three x-intercepts

(c) Symmetric with respect to the x-axis

(d) $(-2, 1)$ is a point on the graph

(e) Symmetric with respect to the origin

(f) Graph passes through the origin